CANONICAL TRANSFORMATION AND THE PROPERTIES OF A MIXTURE OF FERMIONS AND BOSONS AT LOW TEMPERATURE

BY

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DECLARATION

Declaration by the Candidate

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Declaration by Supervisors

This thesis has been submitted for examination with our approval as University Supervisors.

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DEDICATION

To my entire K'Obota family for their invaluable source of motivation and support towards my education.

ABSTRACT

Properties of free and interacting systems of bosons and fermions have been studied theoretically using canonical transformation. In the last few years (1995 to be exact), condensates have been obtained practically in the laboratories for a mixture of bosons and fermions, where optical lattice system is created to trap the mixture and study its properties at very low temperature. Theoretical studies on the properties of such a mixture were undertaken in this work, where a canonical transformation was developed in terms of canonical operators for bosons and fermions. The Hamiltonian H, for the mixture of interacting bosons and fermions was constituted assuming the possible interactions that can exist in the mixture such as boson-boson and boson-fermion interactions. However, the other possible interaction between fermion-fermion is disallowed by the Pauli's principle. The scattering lengths between boson-boson (α_{BB}) and between boson-fermion (α_{BF}) give a measure of interaction strength between the interacting boson-boson and boson-fermion respectively. The values of α_{BB} and α_{BF} vary with the types of bosons and fermions in the mixture and can also be changed by the method of Feshbach resonance. In this thesis, the values of quasi-particle energy of the k^{th} state (E_k) were calculated for different values of α_{BB} and α_{BF} to determine their values for different mixtures. For ${}^{87}_{37}Rb + {}^{40}_{19}K$ mixture with α_{BF} of $150 \times 10^{-8}\ cm,\ 162 \times 10^{-8}\ cm$, $300 \times 10^{-8}\ cm$, and $-209 \times 10^{-8}\ cm$, the corresponding calculated values of E_k were $1.237 \times 10^{-12}\ ergs$, $1.337 \times 10^{-12}\ ergs$, $2.475 \times 10^{-12}\ ergs$, and $-1.103 \times 10^{-12}\ ergs$. Similarly, for ${}^{6}_{3}Li + {}^{7}_{3}Li$ mixture with α_{BF} of $0.2158 \times 10^{-8}\ cm$ had E_k of $5.227 \times 10^{-17}\ ergs$, while for ${}^{23}_{11}Na + {}^{6}_{3}Li$ mixture with α_{BF} of $-1.45 \times 10^{-8}\ cm$ had E_k of $-1.22 \times 10^{-21}\ ergs$. The negative values of E_k mean that the interaction is attractive, where interacting species overlap, whereas positive values of E_k mean that the interaction is repulsive, where the overlap of interacting species reduces.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS AND ABREVIATIONS

SYMBOL	MEANING
a_s	Singlet scattering length
a_k	Old annihilation operator *k-th* state
a_k^+	Old creation operator for the *k-th* state
a_{FF}	Fermion-fermion scattering length
a_{BB}	Boson -boson scattering length
a_0	Bohr's radius
a_{kB}	Annihilation operator for Boson
a_{kF}	Annihilation operator for fermion
m_F	Mass of fermion atoms
m_B	Mass of Boson atoms
m_{BF}	Reduced mass of Fermion-Boson mixture
U_{BB}	Boson-Boson potential or interaction strength
U_{BF}	Boson-Fermion potential or interaction strength
U_{FF}	Fermion -Fermion potential or interaction strength

α_k	New annihilation operator in the k^{th} state
α_k^+	New creation operator in the k^{th} state
α_B	New fermion Destruction operator
$\alpha_{_B}$	New Boson destruction operator
H_F	Energy of Fermion particles
H_B	Energy of Boson particles
k	Boltzmann constant
$\delta\varepsilon_j$	Small variation of Energy in the k^{th} state
δn_j	Small variation of the number of particles
ε_j	Energy of the j^{th} state
ε_k	Quasi-particle Energy of the k^{th} state
E	Hamiltonian
E_B	Energy of Bosons
E_F	Energy of Fermions
${}_2^3He$	Isotope of helium of 3 atomic mass units
$\hbar$	Reduced Plank's constant

λ	Value of Fermi energy
μ	Chemical potential
μ_B	Chemical potential for Bosons
μ_F	Chemical potential for Fermions
N	Total number of particles in a given system
n_i	Number of particles in the i^{h} state
n_B	Number of Bosons in the i^{th} state
n_F	Number of Fermions in the i^{th} state
ω_j	Degeneracy of the j^{th} quantum state
ψ	Wave function of particle
$\lvert\psi\rvert^2$	Atomic density

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CHAPTER ONE

INTRODUCTION

1.1 Background to the study

The field of ultra-cold atomic physics has become sufficiently correlated with condensed matter physics in the last few decades (Bloch *et al*., 2008; Georges, 2007). This combination was made possible by a series of discoveries in cooling methods in the nineties, and this allowed experimentalists to bring a system of large number of atoms into the quantum regime. Some early successes were the observation of Bose- Einstein Condensation (BEC) of ${}^{87}_{37}Rb$ (Anderson, *et al*., 1995), degenerate gas of Fermi atoms of ${}^{40}_{19}K$ (DeMarco *et. al*., 1999) and the BEC of boson ${}^{23}_{11}Na$ (Davis *et al*., 1995). Research was then extended to Fermi degeneracy and boson–fermion quantum degenerate mixtures, especially the study of boson-fermion quantum degenerate mixture of ${}^{6}_{3}Li - {}^{7}_{3}Li$ (Schreck *et al.*, 2001), ${}^{6}_{3}Li - {}^{23}_{11}Na$ (Hadzibabic *et al.*, 2002) and ${}^{40}_{19}K - {}^{87}_{37}Rb$ (Modugno *et al.*, 2002). The simultaneous superfluidity of a boson-fermion mixture was studied at unitarity (scattering length $a \to \infty$) and observed in 2014 (Delehaye, 2016).

1.2 Bose Einstein Condensation (BEC)

An Indian physicist Satyendra Nath Bose studied the statistical Mechanics of different types of particles and developed the Idea of quantum statistical mechanics (Einstein, 1925). Based on the ideas of Bose, Albert Einstein predicted the condensation of bosonic gases, and this called Bose- Einstein -condensation (BEC). The fundamental idea of BEC is that below a certain temperature, called the critical temperature or transition temperature, (Khanna, 1986), the lowest energy state (also called the Zero-momentum-state; ZMS) get macroscopically occupied (Khanna and Mehrotra, 1975; Isaac, *et. al*., 1980). As the temperature of the assembly is decreased, the deBroglie wavelength increases since it varies as $\frac{1}{\sqrt{T}}$. At the critical temperature, T_C, or below, it becomes comparable to the average inter particle separation. At this temperature, the wave functions of the particles get smeared out so that there is sufficient overlap of the wave functions and a BEC is formed. Particles in the assembly must have some inter-particle interaction and any acceptable BEC theory must take this into account, whereas the Einstein prediction applicable to non-interacting particles (atoms) only.

Since 1980, the BEC of dilute gases has been studied experimentally. The first series of experiments were done with Rubidium (Anderson, *et. al*., 1995, Bradley, *et al.*, 1997), Sodium (Davis, *et. al.*, 1995) and Lithium vapours (Schreck, *et. al*., 2001). Laser cooling techniques were developed to obtain BEC in dilute alkali atomic gases (Foot, *et. al*., 1991). In the magneto-optical trap (MOT), the gas is compressed and cooled to micro-Kelvin temperatures using laser cooling techniques. It was found that at a critical phase space density, BEC occurs. In such a condensate come, a macroscopic number of atoms, roughly of order of 10^6, collectively occupy the lowest energy state or ZMS (Kerson and Yang 1957).

1.3 Boson-Fermion Mixture

Liquid ${}_{2}^{4}He$ (Helium), which is a boson, was cooled below its critical temperature at about 5.2 K, and on further cooling it was found that it condenses at about 4.2 K. Kamerlingh Onnes first liquified Helium ${}_{2}^{4}He$ in 1908 (Lee *et al.*, 1957). Further cooling to about $T_C = 2.2K$ (now this temperature is 2.176 K), he noted that the density becomes maximum at this temperature, and then it slightly decreases on further decrease of temperature. This was treated as a phase transition and the liquid helium was named as *He*-4 with very large conductivity. This phase was named as superfluid phase, and was considered as something similar to BEC in the Ideal Bose gas. By treating the system as an assembly of interacting bosons, a number of theories were proposed from time to time (Khanna, 1969) to explain the superfluid behavior of helium ${}_{2}^{4}He$. Later the experimental observation of BEC in dilute atomic vapour of $He-4$ were done and observed. (Anderson *et al*., 1995). As the experimental techniques developed new methods for observing BEC, new techniques were developed for studying the properties of mixtures of boson and fermions, such as ${}_{11}^{23}Na$ - ${}_{19}^{40}K$ (Taylor, *et. al*., 2013), ${}_{11}^{23}Na$ - ${}_{3}^{6}Li$ (Cheng, *et. al*., 2010) and ${}_{37}^{87}Rb$ - ${}_{19}^{40}K$ (Deh, *et. al*., 2021). To study their low temperature behavior, they were cooled to quantum degeneracy temperature (Jee, *et. al*., 2012; Cornell and Wieman, 2002*a*; Dalfovo, *et. al*., 1999). These mixtures were quite stable close to Feshbach resonance of about 100 ms for ${}_{11}^{23}Na$ - ${}_{19}^{40}K$ (Taylor, *et. al*., 2013), and this time is enough to conduct experimental studies of the involved many-body physics, particularly to understand the properties of Bose-Fermi mixture as a function of interaction between bosons and between fermions at temperatures $T \rightarrow 0\,K$. The interaction between fermion is not allowed due to Pauli principle.

Studies have also shown that the mixtures of Bosons and Fermions could collapse when the inter particle interaction between bosons and fermions is attractive. Whereas when the interaction is repulsive, it could lead to a large variety of stable density configurations. (Pethik and Smith, 2002).

1.4 Statement of the Problem

It is now an established fact that BEC of the trapped ultracold atomic gases can be observed experimentally, and this includes pure boson gases, pure fermion gases, and a mixture of boson and fermion gases (Lee, *et. al.*, 2006; Landau, 1933 and Macridui, *et. al.*, 2018). There are a number of heteronuclear combinations of molecules that have been studied experimental, but theoretical studies of such systems have not been fully exhausted. A mixture of bosons and fermions trapped at very low temperature is studied theoretically to understand its properties. The parameters involved are the boson-boson scattering length (a_{BB}), the boson-fermion scattering length (a_{BF}) and the oscillator length of the trapping potential. A model of Hamiltonian is developed for a mixture of bosons and fermions and diagonalized by canonical transformation to obtain the quasi-particle excitation spectrum energy E_k. The dependence of E_k on various parameters was studied to understand the behavior of the system as the parameters are changed.

1.5 Objectives

1.5.1 General Objective

The general objective of this research was to establish the behavior of a mixture of bosons and fermions at cryogenic temperatures.

1.5.2 Specific objectives

The specific objectives were:

i. To derive a canonical transformation that can diagonalize a model Hamiltonian for a boson -fermion mixture.

ii. To apply the derived canonical transformation to the model Hamiltonian and calculate the quasi-particle excitation spectrum energy, E_k, using diagonalized Hamiltonian.

1.6 Justification of the study

The study of interacting boson -fermion mixtures at very low temperatures may at some stage later be of great importance in solving and understanding some of the present unsolved problems in contemporary physics. In solid state physics or in real solid-state materials, how the impurities influence quantum phases and phase transitions is not clearly understood. The many-body quantum phase transition still needs to be well understood. For instance, high-temperature superconductivity is observed in materials that are purposely doped with some impurity atoms, but their exact significance in the mechanism that may be responsible for superconductivity is still not settled (Matus and Bavea, 2017). Now in this case the charge carriers are electrons that are fermions, and if the impurities are bosons, then the whole system will be an assembly of interacting boson-fermion mixture. The theory developed in this thesis could as well be used in future to study the problem of still unsolved high-temperature superconductivity. The crux of the problem is to write the model Hamiltonian.

Another problem of the interacting boson-fermion system is known as the polaron problem (Pitaevskii and Saudro, 2003). The polaron is a bound state between an electron and its induced crystal lattice deformation which is called a phonon. Polaron is also called a self-trapped electron; it is assumed to be trapped in the harmonic potential of the lattice. The quanta of the harmonic potential are bosons. Hence an electron dressed by phonons (polaron) is an example of interacting boson-fermion system. The theory developed in this thesis could as well be used to study the polaron problem (Pethick and Smith, 2002).

A real boson can under some physical process turn into fermions. This is what exactly happens in pair production when a photon (which is a real boson) turns into an electron and a positron which are fermions. Thus, we may have an assembly that is an interacting mixture of bosons and fermions (Molmer, 1998). This problem and some other systems that could be treated as interacting boson-fermion mixtures could be studied by the theory developed in this thesis.

There is more interest in low temperature physics hence this study will contribute to the growing knowledge and literature in this field of research.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

In this chapter, using the principles of thermodynamics, the characteristics of trapped boson-fermion are discussed keeping in mind that the bosons and fermions may be interacting. However, the concept will be valid for a non-interacting system of particles.

For fermions, S-wave interactions are not allowed by Pauli principle because of their half spins. In single-component assembly of ultra-cold fermions and the higher partial wave scattering interaction is energetically not allowed. Statistical thermodynamic behavior can be obtained experimentally for a free Fermi-gas. A similar non-interacting approximation can be obtained for bosons also. The thermodynamics of critical temperature for a weakly interacting Bose gas can also be studied, and such studies will lead to phenomena of condensation in dilute systems at very low temperature.

2.2 Bosons and Fermions

Scientists have been busy studying the stability conditions of interacting mixtures of bosons and fermions, both experimentally and theoretically. The interactions between bosons and fermions could be both repulsive and attractive, and using Feshbach resonance method, the scattering length can be changed from repulsive (scattering length positive) to attractive when the scattering length becomes negative, and vice -versa. The mixture is trapped harmonically since laser beams are used to trap them. (Roth, 2002 and Roth, *et. al*., 2003).

2.3 Statistical Mechanics of Gases

The thermodynamic relations of interest to both theory and experiment are those that will display the occupation of single-particle states and symmetrization characteristics of bosons and fermions. The particle distribution $f(\varepsilon_n)$ is given by (Khanna and Mehrotra, 1975),

$$f(\varepsilon_n) = \frac{1}{\zeta^{-1} e^{\beta\varepsilon_n} + a} \tag{2.1}$$

where $\zeta = e^{\beta\mu}$ is the fugacity, a parameter which is very often used instead of the chemical potential μ; $\beta = 1/kT$ is a measure of temperature of the assembly. To get the occupation number of the system, Maxwell-Boltzmann, Bose-Einstein and Fermi Dirac statistics, the quantity $'a'$ in equation (2.1) has numerical values given as:

$$a = \begin{cases} 0 & - \quad Maxwell-Boltzmann-statistics \\ -1 & - \quad Bose-Eistein-statistics \\ +1 & - \quad Fermi-Dirac-Statistics \end{cases} \tag{2.2}$$

Due to Pauli exclusion principle, occupation number of a single-particle state for fermions in the Fermi Dirac Statistics is only one.

For bosons, since $a = -1$, the denominator can be zero, and hence occupation number f can become infinity, and this is BEC.

The thermodynamic relationships can be introduced which are relevant for harmonically trapped gases, where as in the simplest case of the homogeneous gas, it is a simple relation. Generally, the three dimensional (3D) harmonic oscillator potential has the form:

$$V(\vec{r}) = \frac{1}{2}(m\omega_1^2 x_1^2 + m\omega_2^2 x_2^2 + m\omega_3^2 x_3^2) \quad (2.3)$$

The thermodynamic properties can be calculated by transforming from discrete energy level scheme to continuous energy level scheme. However, the ground state for an assembly of bosons is to be excluded from integration.

As a function of energy ε, the density of state is written as (Roth, 2002 and Roth and Feldruieier, 2003).

$$g(\varepsilon) = \frac{\varepsilon^2}{2(\hbar\varpi)^3} \quad (2.4)$$

In the case of bosons, the ground state can have very large number of particles, i.e., it can be occupied macroscopically, whereas in the case of fermions in the ground state, each state can have only one particle due to Pauli principle.

The number of particles in the excited state, N_{ex} can be calculated by writing,

$$N_{ex} = \int_0^\infty f(\varepsilon)g(\varepsilon)d\varepsilon \quad (2.5)$$

Substituting for $g(\varepsilon)$ from Eq. (2.4) in Eq. (2.5), we can calculate the number of particles N_{ex} as,

$$N_{ex} = -a\left(\frac{k_b T}{\hbar\varpi}\right)^3 Li_3(-a\zeta) \quad (2.6)$$

where n *is* a positive integer from Eq. (2.6) is a class of hypergeometric function with characteristics such that,

$$-Li_n(-\zeta) = f_n(\zeta) \tag{2.7}$$

A plot of $f(\varepsilon)$ against E_F is shown in Fig. 2.1

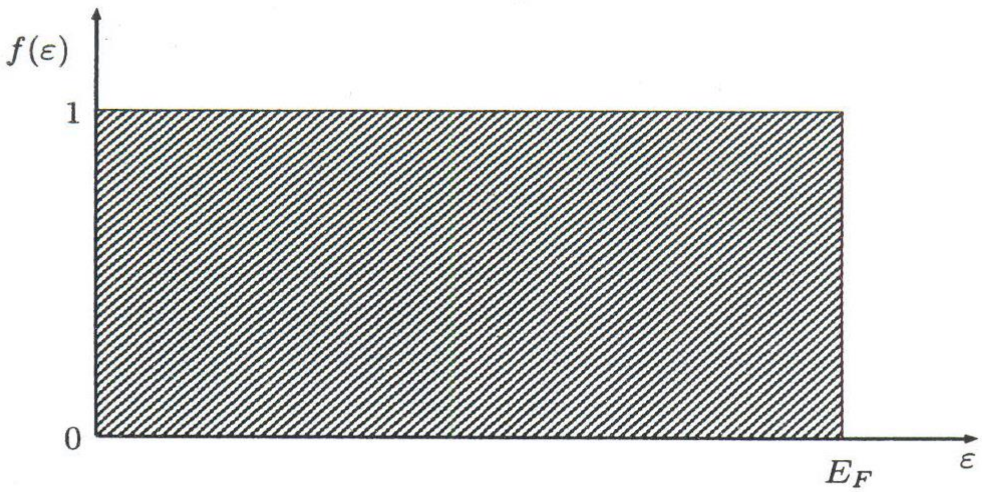

Figure 2.1: Zero-temperature Fermi Distribution (Roth, 2002 and Roth and Feldruieier, H, 2003).

Therefore, $Li_n(\zeta)$ in Eq. (2.7) is then given as,

$$Li_n(\zeta) = g_n(\zeta) \tag{2.8}$$

and $g_n(\circ)$ is the Bose-Einstein function and can be written as a series, i.e.,

$$Li_n(y) = \sum_{l=1}^{\infty} \frac{y^l}{l^n} \tag{2.9}$$

In order to get the value of N_{ex} in Eq. (2.6) the value of N_{ex}, can be evaluated (Roth, 2002 and Roth and Feldruieier, 2003). using the following integral:

$$\int_0^{\infty} \frac{x^{(n-1)}}{\zeta^{-1}e^x+1}dx = -\Gamma(n)Li_n(-\zeta) \tag{2.10}$$

2.3 Fermi Energy in Fermi -Dirac Statistics

For Fermi-Dirac case, $a=+1$ from Eq. (2.2) and single-state occupation is one only due to Pauli-principle such that:

$$f(\varepsilon)=\frac{1}{e^{\frac{\varepsilon-\mu}{kT}}+1} \tag{2.11}$$

At very low temperature, T , ε is close to Zero, or $\varepsilon<\mu$ and hence $f(\varepsilon)$ becomes 0 or 1 on integrating Eq. (2.5) yields,

$$f(\varepsilon)=\begin{cases}1 & for \quad \varepsilon<E_F \\ 0 & for \quad \varepsilon>E_F\end{cases} \tag{2.12}$$

Therefore,

$$N=\int_0^{\infty} f(\varepsilon)g(\varepsilon)d\varepsilon=\int_0^{E_F} g(\varepsilon)d\varepsilon=\frac{E_F^3}{6(\hbar\varpi)^3} \tag{2.13}$$

From Eq. (2.13), we can write,

$$E_F=\hbar\varpi(6N)^{1/3} \tag{2.14}$$

And the Fermi temperature T_F is given by,

$$T_F=\frac{\hbar\varpi}{k_B}(6N)^{\frac{1}{3}} \tag{2.15}$$

since $E_F=k_BT_F$.

If in trap the number of fermions $N = 10^6$ (Khanna, Das, and Sinha (1969), and $\varpi = 2\pi \times 50Hz,$ then $T_F = 0.26\mu K$. Eliminating particle numbers in Eq. (2.6) gives the following results of thermodynamic importance for an assembly of trapped fermions.

$$Li_3(-\zeta) = \frac{-1}{6(T/T_F)^3} \tag{2.16}$$

2.4 Theory of Bose-Einstein condensation

In the case of bosons for $T > 0K$, the excited states can accommodate N_{ex} particles where,

$$N_{ex} = \left(\frac{k_b T}{\hbar\varpi}\right) g_3(1) \tag{2.17}$$

The remaining particles will be in the ground state such that,

$$N = N_0 + N_{ex} \tag{2.18}$$

Where N_0 equals to the number of the particles in the ground state or ZMS, and this number is very large such that $N_0 \cong N$. This phenomenon of macroscopic occupation of the ground state is known as Bose-Einstein condensation Khanna. (1969). The temperature of condensation can be obtained from Eq. (2.17) such that:

$$T_C = \frac{\hbar\varpi}{k_B}\left(\frac{N}{g_3(1)}\right)^{\frac{1}{3}} \tag{2.19}$$

For the trap with 10^6 atoms (Khanna, Das, and Sinha, 1969) and $\varpi = 50Hz, T_C = 226nK$. As a result of the interactions among particles, there will be small corrections to the critical temperature (Roth, 2002). In the condensed state, an important quantity is the condensate fraction n given by (Roth and Feldruieier, 2002).

$$n = \frac{N_0}{N} = 1 - \left(\frac{T}{T_C}\right)^3 \quad (2.20)$$

At the temperature $T = T_C, n = 0K$, which means that there are hardly any particles in the condensed state or ZMS. The relation between T_C and is $T_F$of the form (Roth & Burnett, 2003).

$$\frac{T_C}{(N_B)^{\frac{1}{3}}} \cong 0.40 \frac{T_F}{(N_F)^{\frac{1}{3}}} \quad (2.21)$$

In a typical mixture of ${}^{87}_{37}Rb$, boson and ${}^{40}_{19}K$, fermion, the number of bosons N_B is more than the number of fermions N_F. Thus, we can conclude from Eq. (2.21) that, in general, $T_C > T_F$.

2.5 Bose -Einstein condensate (BEC) in a wave form

BEC refers to a macroscopic occupation of the ground state of a weakly interacting assembly of bosons. If N_0 is the number of particles in the condensate state and ψ represents its microscopic wave function (Khanna, & Phukan (1972) then,

$$N_0 = |\psi(r)|^2 \quad (2.22)$$

2.6 Interacting mixtures

So far, the behavior of degenerate bosonic and fermionic gases has been discussed. Now the properties of interacting mixtures of bosons and fermions are discussed. When bosons and fermions exist simultaneously in a harmonic or optical trap, simultaneous degeneracy of the mixture can be achieved, and the interactions between the particles of both the gases can fundamentally affect the behavior och waves then analyses for a BEC in sinusoidal external potential between bosons and fermion (Pethik and Smith, 2002). A lot of work were done theoretically as was documented (Rogel, 2001).

The behaviour of the mixture of bosons and fermions can be uf the mixture. The interacting boson ns represented by scattering length a_{BF} strongly affect the density profile of bosonic and fermionic clouds depending upon the strength and sign of a_{BF} , whether the interaction is attractive or repulsive, phase separations can occur (Modugno, *et. al*., 2002 mixtures or boson-fermion mixtures in a trap are of great importance. Such mixtures can have many phases, and the trapped Bose-fermion and were first analyzed, Experiments were done using Blonderstood by writing modified system of equations (Minguzzi and Tosi, 2000; Bransde and Joachim, 1989 and Goldwin, 2002). One of these equations is written as:

$$\left[-\frac{\hbar^2}{2m_B}\Delta + V_B(\tilde{r}) + g_{BB}n_B(\tilde{r}) + g_{FB}n_F(\tilde{r})\right]\psi = \mu_B\psi \qquad (2.23)$$

Where $V_{B}(r)$ is the potential for bosons and $V_{F}(r)$ is the potential for Fermions. The second equation is…

$$n_F(\vec{r}) = \frac{(2m_F)^{\frac{3}{2}}}{6\pi^2\hbar^3}\max[\mu_F - V_F(\vec{r}) - g_{FB}n_B(\vec{r})]^{3/2} \tag{2.24}$$

Where $n_{F(r)}$ is the density of fermions and $n_{B(r)}$ is the density of Bosons.

The coupling parameters are defined by,

$$g_{BB} = 2\pi\hbar^2 a_{BB} / m_{BB} \tag{2.25}$$

$$g_{FB} = 2\pi\hbar^2 a_{FB} / m_{FB.} \tag{2.26}$$

Here m_{BB} and m_{FB} are the reduced masses for boson-boson and fermion-boson interactions respectively. The Thomas-Fermi approximation leads to the bosonic and fermionic equations such that for $n_{B(r)}$ and $n_{F(r)}$ we get,

$$\left.\begin{aligned} n_B(\vec{r}) &= \frac{1}{g_{BB}}\max[m_B - g_{FB}.n_F(\vec{r}) - V_B(\vec{r})] \\ n_F(\vec{r}) &= \frac{(2m_F)^{\frac{3}{2}}}{6\pi^2\hbar^3}\max[m_F - V_F(\vec{r}) - g_{FB}.n_B(\vec{r})]^{\frac{3}{2}} \end{aligned}\right\} \tag{2.27}$$

The particle number N_B for bosons and N_F for fermions fix the corresponding chemical potentials such that,

$$N_B = \int_0^{\infty} n_B(m_B, \vec{x})d^3x \tag{2.28}$$

$$N_F = \int_0^{\infty} n_F(m_F, \vec{x})d^3x \tag{2.29}$$

If the mixture of ${}^{40}_{19}K$ and ${}^{87}_{37}Rb$ is trapped magnetically then trapping potential (Ospelkaus, *et. al*., 2006) are

$$V_{Rb}\left(\vec{r}\right)=V_K\left(\vec{r}\right) \tag{2.30}$$

However, the trapped frequency ratio is,

$$\omega_K/\omega_{Rb}=\sqrt{m_{Rb}/m_K}=\sqrt{87/40} \tag{2.31}$$

And coordinates can be rescaled as,

$$\tilde{x}_i=\sqrt{\frac{m_F}{2}}\omega_{F,i}.x_i \tag{2.32}$$

In this case, $\tilde{r}^2=V_B=V_F$, where $\tilde{r}^2$ represent the potential energy of the mixture. Thus, the coupled Thomas-Fermi problem becomes;

$$n_B(\vec{r})=\frac{1}{g_{BB}}\max[m_B-g_{FB}.n_F(\vec{r})-\vec{r}^{\,2}] \tag{2.33}$$

$$n_F(\tilde{r})=\frac{(2m_F)^{3/2}}{6\pi^2\hbar^3}\max[m_F-\tilde{r}^2-g_{FB}.n_B(\tilde{r})]^{3/2} \tag{2.34}$$

These values depend on $\tilde{r}=\sqrt{(\tilde{x}_1^2+\tilde{x}_2^2+\tilde{x}_{31}^2)}$. In these set of coordinates, Eq. (2.28) gives,

$$N_B=\left(\frac{2}{m_B}\right)^{3/2}\frac{1}{(m_B)^3}\int_0^\infty 4\pi^2\tilde{r}^2 n_B(\tilde{r})d\tilde{r} \tag{2.35}$$

And Eq. (2.29) gives,

$$N_F = \left(\frac{2}{m_F}\right)^{3/2} \frac{1}{(m_F)^3} \int_0^\infty 4\pi^2 \tilde{r}^2 n_F(\tilde{r}) d\tilde{r} \qquad (2.36)$$

where the spherical symmetry has been used in the problem in the new set of coordinates. It is to be noted that the above problem is formulated solely in terms of potential energy $\vec{r}^2$ of the mixture, and the trapping potential enters the calculation only through the geometric mean trapping frequency $\varpi = \sqrt[3]{\omega_1 \omega_2 \omega_3}$. The fact that in the calculations the mean trapping frequency is used so that in different experiments becomes relevant and easy under certain conditions and the contemplated values of μ_F, N_F can be written as (DeMarco, *et. al*., 2001),

$$N_F = \frac{4}{(\hbar \varpi_F)^3} \int_0^\infty d\tilde{r} \max[m_F - \tilde{r}^2 - g_{FB} n_B(\tilde{r})]^{3/2} \tilde{r}^2 \qquad (2.37)$$

In Eq. (2.37), μ_F is treated as Fermi energy and using the process of iteration results in the right-hand side of it is equal to N_F. Similarly, the value of N_B is written as (Ospelkaus, *et*. *al*., 2006(a),(b):

$$N_B = \left(\frac{4\pi}{g_{BB}}\right) \frac{4^3}{(m_F^2 \varpi_F)^3} \int_0^\infty d\tilde{r} \max[m_B - \tilde{r}^2 - g_{FB} n_F(\tilde{r})]^{3/2} \qquad (2.38)$$

The value of the chemical potential μ_B is fixed by solving Eq. (2.38), lead to the following possible results,

I. Bosonic density converges (means finite), and fermionic density is zero at the centre of the trap this is a case of phase separation.

II. Both boson (N_B) and fermion (N_F) values can diverge and lead to collapse

III. Both densities can converge (have finite values) as in the self-consistent field approximation.

IV. By fixing values of $(N_B),(N_F)$ and ω, a_{BB}, a_{BF} etc., the above-mentioned phases can be realized experimentally by the process of Feshbach resonance. The existence of phases will be determined by coupling ratio, γ such that,

$$\gamma = \frac{g_{FB}}{g_{BB}} \tag{2.39}$$

2.7 Non interacting Assembly of Bosons and Fermions

When there is no interaction between bosons and fermions in an assembly of mixture, then $g_{FB} = 0$, and hence $\gamma = 0$ in Eq. (2.39). In such a case, the clouds of bosons and fermions move independently of each other. As we switch on interaction slowly, the value of γ will change and this variation can be used to study the effect of one species on the other as γ changes. The switching on of interaction will lead to change in density profile of bosons and fermions in the limit $T \rightarrow 0K$. Such changes and the (BEC) are shown in Figure 2.2.

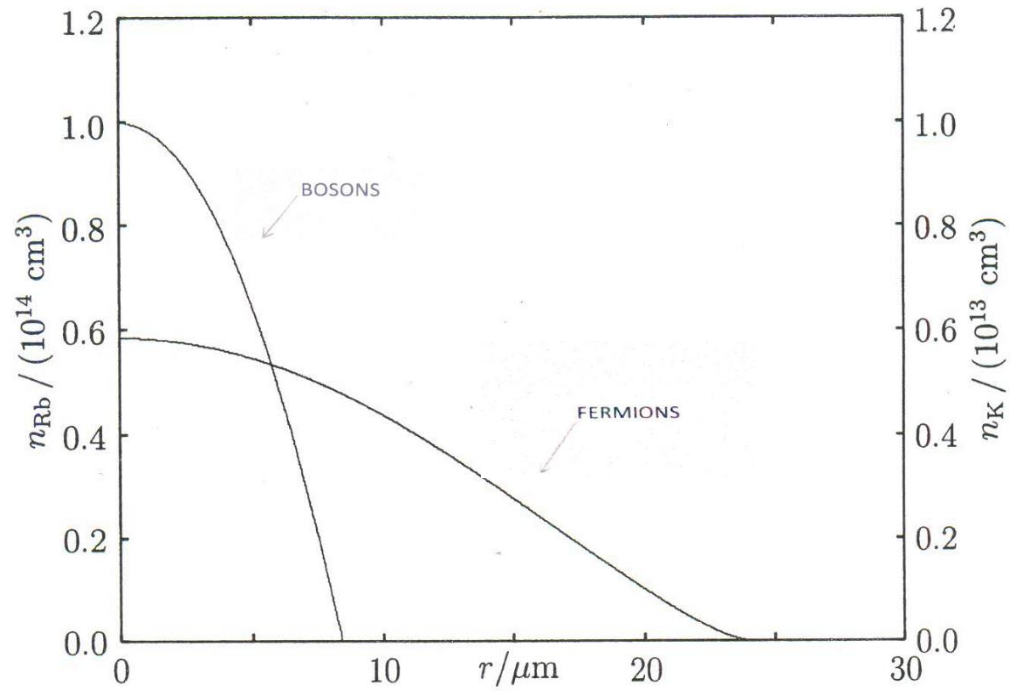

Figure 2.2: Thomas Fermi model Display of non-interacting Bose-Fermi mixture (Star, *et. al*., 2004).

The particle density of the BEC at the centre of the trap is around $10^{14}cm^{-3}$ and that of Fermionic gas is around $0.6\times10^{14}cm^{-3}$ (it is diluted). Simultaneously, the spatial extension of Fermi cloud is very large compared to the size of BEC condensate. The spatial extension of BEC is of order of $8\mu m$ for boson cloud and that of Fermion cloud it is around $23\mu m$.

Changes in interaction between the bosons and fermions represented by variation in g_{FB} affect the density profiles of bosons (n_B) and fermions (n_F). This leads to changes in the mean field potentials, $g_{FB}n_B$ for boson, and $g_{FB}n_F$ for fermions.

2.8 Attractive Interactions between Bosons and fermions in a trap

If the interaction between the bosons and fermions is attractive, say for $\gamma = -4$, the density profile for bosons and fermions in the trap is displayed as shown in Figure 2.3 (Star, *et. al.*, 2004). Up to a distance of about $9\mu m$ there is sufficient overlap of bosons and fermions. This would mean that $g_{FB} n_F \cong g_{FB} n_B$ (Roth, 2002 and Roth and Feldruieier, 2003). But the fermion cloud is spread out beyond this resulting in phase separation. It is clear that the stronger Bose-Fermi attraction greatly affects the density profiles n_B and n_F as shown in the Figure 2.3.

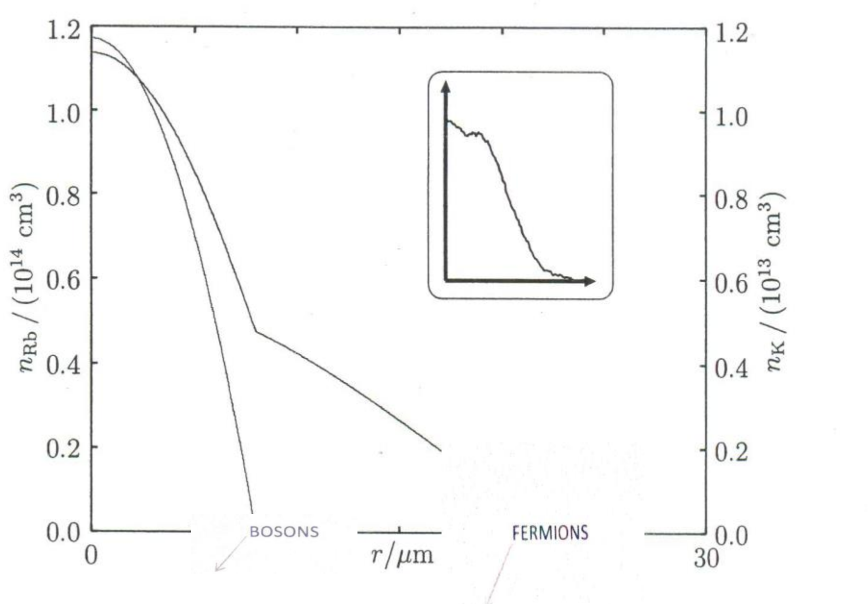

Figure 2.3: Mixture of boson and fermion with smaller attractive interaction (Star, *et. al.*, 2004).

2.9 Role of Repulsive heteronuclear Interactions in the Boson-fermion mixture in the trap

When the heteronuclear interaction between bosons and fermions is weak $(\gamma \cong 4)$, The fermion density n_F in the centre of trap is large, and the boson density n_B is spread out as shown in Figure 2.4. This leads to phase separation.

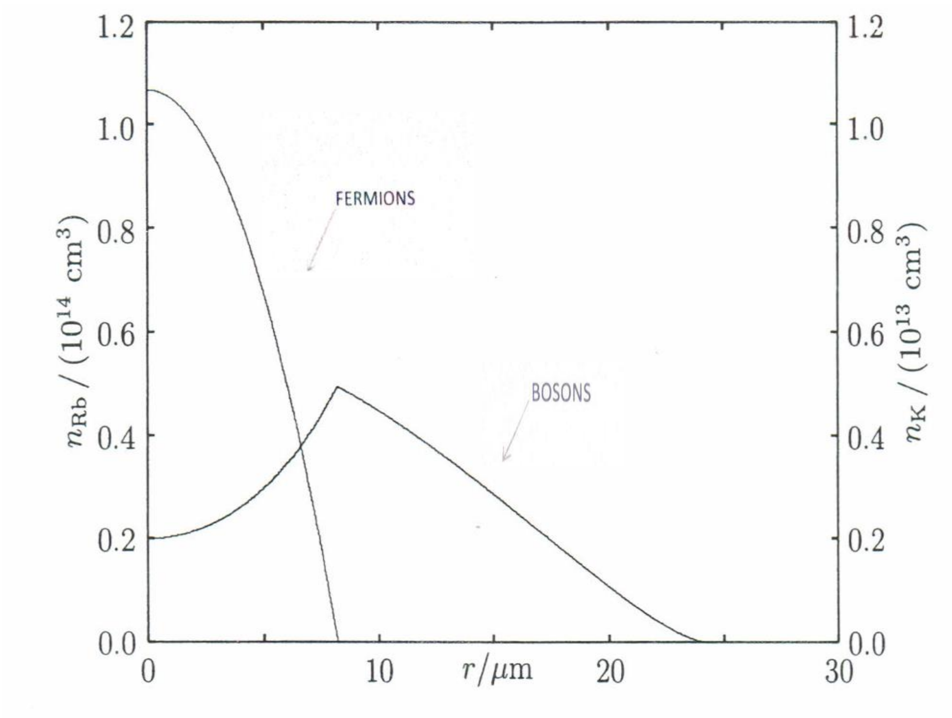

Figure2.4: Mixture of Bosons and fermions with slightly repulsive interaction in the trap (Star, *et. al*., 2004).

2.10 Experimental and Theoretical information about Boson-Fermion mixtures

Very impressive results, both experimentally and theoretically, have been obtained in the past about the boson-fermion quantum degenerate mixtures trapped by optical and magneto-optical trapping methods (Will, 2013; Santos, *et. al*., 2004).

Pauli blocking of collisions was also observed experimentally (Will, 2013). Boson-Fermion quantum degenerate mixture of the ${}^{6}_{3}Li/{}^{7}_{3}Li$, ${}^{6}_{3}Li/{}^{23}_{11}Na$ and ${}^{87}_{37}Rb$ / ${}^{40}_{19}K$ studies have been done. Quantum degenerate fermion mixtures in optical lattice potentials (Mathey, *et. al.*, 2004) and also the interacting mixtures of bosons and fermions in optical lattice potentials have been studied to see how phase separation can occur (Bose, 1924 and Luhmann, *et. al*., 2009).

The important parameters governing the Physics of interaction of the boson-fermion mixtures, are the scattering length a_{BB} for boson–boson interaction, and a_{BF} boson-fermion scattering lengths. Large values of (***a***) in the quantum degenerate state will correspond to large effective interaction. Component phase separation occur for $a > 0$ (repulsive interactions) (Rogel, 2001) and stable mixture can be obtained for $a < 0$ (attractive interaction) (Modugno, *et. al*., 2002). In the trap, atomic densities n_B and n_F of the components play an important part in determining phase separation and stability of the mixtures. Temperature in the trap could be as low as $2mK$, and the density of the gases in the mixture could vary substantially. They may lie between $n_0 \cong 3\times10^9 cm^{-3}$ to $n_0 \cong 10^{13} cm^{-3}$ (Khanna, & Phukan (1972). Similarly, the scattering length a_{BB} and a_{BF} will depend on the density of the components and the temperature of the trap. They also depend on the depth and width of the trapping potential. The values of the scattering lengths a_{BF} can be changed from positive (repulsive) to negative (attractive) and vice-versa.

Total density (mass density) of the mixture is given by ρ i.e $\rho = \rho_B(T) + \rho_F(T)$ where $\rho_B(T)$ the density of bosons is and $\rho_F(T)$ is the density of fermions at any temperature (T).

2.11. Characteristics of Superfluidity

The non-zero superfluid mass density (Cramer, 2011) is the defining characteristic of super fluids;

1. Landau criteria for superfluid are neither necessary nor sufficient.
2. Density $\rho = \rho_n(T) + \rho_s(T)$ [Two -Fluid model] where $\rho_n(T)$ is the density of normal component with velocity V_n ;and $\rho_s(T)$ is the velocity of superfluid component with velocity V_s .
3. Mass current $= \rho_s V_s + \rho_n V_n$.
4. Entropy current $= SV_n$ (carried by normal fluid only)
5. Superfluid density and condensate density are different: $\psi(x)$ = Order parameter, $n_0 =$ condensate density $= |\psi|^2$, where ψ is the wave function of the macroscopic ground state.

Interactions between the particles in the ground state (ZMS) can excite the particles into excited finite momentum single particle states. In ${}^4_2He$, at $T = 0K$, $\rho_s / \rho = 1$ and [⟨10%] of the particles are in condensate (Bruderer, *et. al.*, 2008). In superfluid $n_0 = 0$ at $T = T_C$. Near T_λ, $\frac{\rho_s}{\rho} \cong (T_\lambda - T)^\alpha$, $\alpha = 0.67 \pm 0.03$. According to Landau Criteria for superfluidity (Gunter, *et. al*., 2006), $V_{crit} \approx 60 m/s$ for superfluid ${}_2^4He$. Landau Velocity, V_{crit}, for superfluidity means that if the superfluid tends to move with a velocity, V, greater than V_C, superfluidity is destroyed (Cornell and Wiemann, 2002*b*).

2.12 Landau Criteria for Super fluidity

According to the Landau criteria (Buonsante, *et. al*., 2008), the critical velocity in a single component condensate is given by the expression,

$$V_c = \min\left[\frac{E_0(k)}{\hbar k}\right] \tag{2.40}$$

where $E_0(k)$ is the excitation spectrum in an immovable condensate and k is the wave vector. Eq. (2.40) can be applied to a two-component condensate only in the case when both components move with the same velocity.

The elementary excitation energy of the two components will depend on the velocities V_1 of one component and V_2 of the second component. Since energy is a scalar quantity, the total energy of the two-component will be simply the algebraic sum of the energies of each component. This is also known as the principle of positivity of energies of elementary excitations. However, if we calculate the quasi-particle energy $E_0(k)$ of the two component Bose gases in the condensate state, then the condition that the velocities V_1 and V_2 of the two component Bose gases must be the same can be discarded, and Eq.(2.40) can be used to get the Landau's critical velocity for a stable superfluid two-component mixture of Bose gases. In such a case, the expression $E_0(k)$ will be the quasi-particle energy of two component mixture of Bose gases in the superfluid (condensate) state. If m_1 is the mass of particle of one component and m_2 is the mass of particle of the second component, and a_{12} is the scattering length for the contact scattering, then the scattering potential energy will be g_{12}, such that,

$$g_{12} = \frac{2\pi\hbar^2 a_{12}(m_1 + m_2)}{m_1 m_2} \tag{2.41}$$

The transition temperature T_C for Bose -Einstein Condensation is given by

$$T_C = \frac{3.31\hbar^2 n^{2/3}}{4\pi^2 km} \tag{2.42}$$

where $n =$Critical particle number density$= \frac{N}{V}$, and N=Total number of particles in the volume V.

The temperature T_C is the transition or critical temperature at which the addition of more particles leads to BEC or the superfluid state. Since T_C depends on the particle number density n, and the mass m of the particles for a two -component state, T_C will be different for each component. For the same value of n, T_C will be lower for higher value of m. Thus, if $m_2 > m_1$, then $T_{C2} < T_{C1}$, and the mixture will be superfluid if $T \le T_{C2}$. However, if $T > T_{C2}$, but $T < T_{C1}$, then the component with particle mass m_1 will be superfluid, and the component with mass m_2 will not be superfluid. This can lead to drag of component with mass m_1 and ultimately disappearance of superfluidity of the component with mass m_2.Thus, for two-component system to be in the superfluid state, $T \leq T_{C2}$. Thus, demixing of the superfluid mixture can take place by changing T_C.

Now the quasi-particle energy excitation spectrum for each component maybe different and this will mean that the Landau Critical velocity for the superfluidity of each component will be different. If, however, we obtain the quasi-particle energy $E_0(k)$ of a system of two-component interacting mixture of bosons, we can get a unique value for the Landau's Critical velocity for the superfluidity of the mixture. Similar criteria will apply to a superfluid mixture of bosons and fermions.

CHAPTER THREE

THEORETICAL DERIVATIONS

3.1 Introduction

There have been a large number of attempts, both theoretical and experimental, to study the properties of an interacting mixture of bosons and fermions (Cramer, 2011; Buonsante, 2008 and Regal, *et. al*., 2003). Quantum many-body theories have been used to study the properties of such systems, especially the role of scattering length, the optical potential well depth, the potential well width, and the particle number density of bosons and fermions in the interacting mixture. Another aspect that has been extensively studied was the role of attractive and repulsive interactions on phase separation.

In most of the theories developed to study the properties of interacting boson-fermion mixtures in the optical lattices (Cramer, 2011; Gunter, *et. al*., 2006 and Regal, *et. al*., 2003), a model Hamiltonian is written, and then the energy excitation spectrum is obtained in terms of parameters that affect the physical properties of the many body system. The existing theories depend on Bogoliubov canonical transformation for interacting bosons and fermions to finally obtain the quasi-particles spectrum.

To my knowledge, there is as yet no canonical transformation for an assembly of interacting bosons and fermions that is defined by the bosons-fermions scattering length a_{BF} . There are, however, canonical transformations for free bosons, free fermions, interacting bosons and interacting fermions (Khanna, 1986; Khanna and Mehrotra, 1975).

There had been successful experimental observation of the (BEC) of the ultracold atomic gas (Cubizolles, 2003), and then a progress (Jochim, *et. al*., 2003) has been made in the physics of quantum gases, and this includes mixtures of boson-boson, fermion-fermion and boson -fermion gases. Molecular formations have been performed experimentally in ultra-cold gases for two fermions ${}^{40}_{19}K$ (Köhl, *et. al*., 2005; Wynar, 2000; Herbig, *et. al*., 2003 and Greiner, *et. al*., 2002) two bosons (${}^{87}_{37}Rb$)(Mandel, *et. al*., 2003 and Batroni,1990) and boson fermion heteronuclear molecules; ${}^{87}_{37}Rb$ - ${}^{40}_{19}K$ (Star, *et. al*., (2004). Thus, many experimental and theoretical works have been done, for instance, on the superfluid-Mott insulator transition in bosons (Jaksch, *et. al*., 1998; Jack and Yamashita, 2003; Roati, *et. al*., 2004; Fujihara, *et. al*., 2007 and Miyawaka and Meystre, 2006) fermionic (Morales, *et. al*., 2009 and Bogoliubov, 1995) and boson-fermion system (Khanna,1969 and Khanna, 2001).

Many-body quantum calculations based on microscopic model have been done for boson-boson, fermion-fermion and boson -fermion mixtures, and calculations could include inter-particle interaction, especially atom-molecule and molecule-molecule ones. Earlier systems of free fermions, free bosons were studied using the Bogoliubov canonical transformation (Khanna, 1986; Khanna and Mehrotra, 1975 and Bogoliubov, 1995). It was used to approximately describe the system in terms of elementary excitations or quasi-particles. As a rule, the canonical transform serves to define quasi-particles. The canonical transformation is used to diagonalize the actual Hamiltonian for bosons and in the end, we obtain the energy excitation spectrum E_K for the quasi-particles. In the case of fermions, however, we obtain the value of E_K and also the expression for the energy gap Δ for the superconducting solution. All this exercise is meant for an assembly of pure bosons or an assembly of pure fermions of the same atomic structure, i.e, identical and indistinguishable particles.

There have been many successful experiments to trap and cool bosonic and fermionic species that have been reported. Ground state properties of a mixture of N_B bosonic and N_F fermionic atoms in an external trapping potential at zero temperature have been studied. The atoms are considered as inert interacting bosonic or fermionic particles. There could also be interaction between bosons and fermions. Since they (particles) are in a trapping potential, the system will be dilute mixture of bosons and fermions. One could also study a system with a very large number of bosons and fermions. In either case it will be a many-body problem of interacting bosons and fermions such that the total number of particles can be written as

$$N = N_B + N_F \tag{3.1}$$

The Hamiltonian H of a binary boson-fermion mixture can be written as,

$$H = H_B + H_F + H_{BF} \tag{3.2}$$

Where the H_B involves only the bosonic component, H_F describes only the fermionic component, and H_{BF} describes the interactions between two species.

+For a pure boson, or a pure fermion system, the Hamiltonian can be written in terms of creation and annihilation operators, then the Bogoliubov canonical transformation can be used to diagonalize the Hamiltonian to get the energy excitation spectrum E_K. For the Hamiltonian given by Eq. (3.2) which stands for a mixture of bosons and fermions, we need a canonical transformation that should be applicable to such a mixture, and this canonical transformation is derived in section 3.2 (Roth, *et. al*., 2001). This transformation can be used to transform the model Hamiltonian H for a mixture of interacting bosons and fermions, and finally to get the quasi-particle energy of the mixture.

3.2 Canonical Transformation for an interacting mixture of bosons and fermions

In developing this canonical transformation, the boson creation (a_B^+) and annihilation operators (a_B) are to be combined with the fermion creation operator (a_F^+) and the fermion annihilation operator $\left(a_F\right)$. The new transformation operators, say represented by α will contain a_B's and a_F's. The new operators $\alpha's$, will be used to diagonalize the model Hamiltonian (H). The new operators α and the old operators a_B and a_F are combined in a transformation keeping in mind the commutation laws for boson operators, and the anti-commutation laws for fermion operators.

First, we consider a mixture of free bosons and free fermions. If a_k and a_k^+ are old annihilation and creation operators, respectively, for the k^th^ state, α_k and α_k^+ are new annihilation and creation operators for the kth state, then for fermions they are related to each other by the following transformation,

$$\alpha_k = \begin{cases} a_k & for \quad \varepsilon_k \rangle \lambda \\ a_{-k}^{+} & for \quad \varepsilon_k \langle \lambda \end{cases} \tag{3.3}$$

Where ε_k refers to the energy of the k^{th} energy level and λ is the value of Fermi energy, i.e., $\lambda = \varepsilon_F$.

Here the old operators $a_k's$ satisfy anti - commutation relations, and it is easy to verify from equation Eq. (3.3) that the $\alpha_k's$ also satisfy the anti-commutation relations, and hence the transformation given by this equation is a canonical transformation. From Eq. (3.3), we can write

$$a_k^{+} a_k = \begin{cases} \alpha_k^{+}\alpha_k & for \quad \varepsilon_k \rangle \lambda \\ \alpha_{-k}\alpha_{-k}^{+} = 1 - \alpha_{-k}^{+}\alpha_{-k} & for \quad \varepsilon_k \langle \lambda \end{cases} \tag{3.4}$$

The canonical transformation in Eq. (3.3) brings us to a new description of the free fermion system in terms of quasi- particles of which the creation and annihilation operators are α_k^{+} and α_k, respectively. The transformation shows that for $\varepsilon_k > \lambda$, a quasi-particle of momentum k in the new description corresponds to an ordinary particle of momentum k in the old description. But for $\varepsilon_k < \lambda$, a quasi-particle of momentum k in the new description corresponds to a hole of momentum $(-k)$ in the old description.

For free bosons, there is nothing like fermi energy (Khanna,1986 and Khanna, 1969). Thus, the new and old operators will be related as,

$$\alpha_{kB} = a_{kB} \tag{3.5}$$

$$\alpha_{kB}^{+} = a_{kB}^{+} \tag{3.6}$$

Thus, in a mixture of bosons and fermions, the energy levels below ε_F will be occupied by fermions only whereas the energy levels above ε_F will be occupied by both bosons and fermions. This is a direct consequence of Pauli exclusion principle that forces the fermions to occupy 'excited' single - particle states, whereas the bosons in a BEC will occupy the ZMS and other exited states (Khanna and Mehrotra 1975).

Energy levels against occupation by both bosons and fermions as shown in the Figure 3.1 below

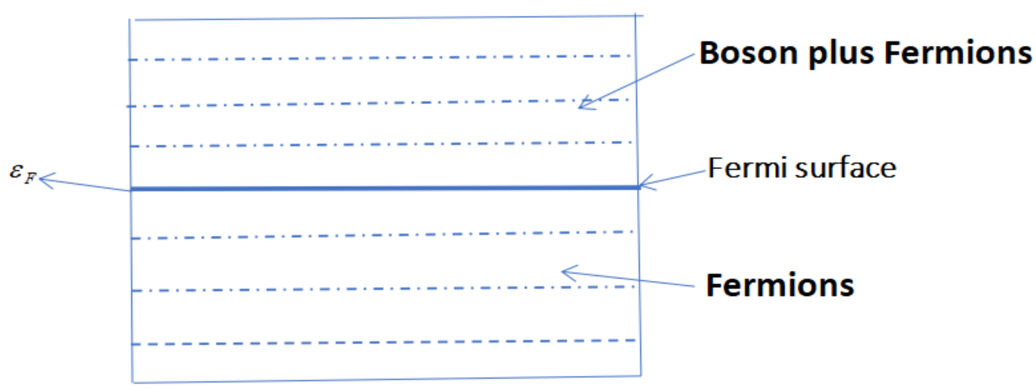

Figure 3.1: The occupation of atoms above and below the Fermi surface.

When the temperatures are very low, most of the fermions will fill the levels below ε_F , but a few may stray to levels above ε_F as illustrated in Figure 3.1. For bosons, ε_F will be treated as the ZMS. Fermions that are locked in the energy levels below ε_F may play no role in determining properties of the mixture of bosons and fermions. Consequently, we can write the new operator for a mixture of free bosons and fermions as,

$$\alpha_k = \alpha_{kB} + \alpha_{kF} = a_{kB} + a_{kF} \tag{3.7}$$

For the transformation in Eq. (3.7) to be canonical, α_k 's and a_k 's must satisfy the same commutation or anti-commutation laws. Here a_{kB} is the annihilation operator for bosons and will thus satisfy commutation laws, whereas a_{kF} is the annihilation operator for fermions and will thus satisfy anti- commutation laws hence we can write,

$$\{a_{kF}, a^{+}_{k'F}\} = \delta_{kk'} = a_{kF}a^{+}_{k'F} + a^{+}_{k'F}a_{kF} \text{ (Anti - commutation)} \tag{3.7a}$$

$$[a_{kB}, a^{+}_{k'B}] = \delta_{kk'} = a_{kB}a^{+}_{k'B} - a^{+}_{k'B}a_{kB} \quad \text{(Commutation)} \tag{3.7b}$$

Now from Eq. (3.7) it can be written that,

$$\left.\begin{aligned} \alpha_k &= a_{kB} + a_{kF} \\ \alpha^{+}_k &= a^{+}_{kB} + a^{+}_{kF} \end{aligned}\right\} \tag{3.8}$$

$$\{\alpha_k, \alpha^{+}_{k'}\} = \delta_{kk'} = \alpha_k \alpha^{+}_{k'} + \alpha^{+}_{k'}\alpha_k \tag{3.9}$$

Substituting from Eq. (3.7) and Eq. (3.8) in Eq. (3.9) yields,

$$\begin{aligned} \{\alpha_k, \alpha^{+}_{k'}\} = \delta_{kk'} &= (a_{kB} + a_{kF})(a^{+}_{k'B} + a^{+}_{k'F}) + (a^{+}_{k'B} + a^{+}_{k'F})(a_{kB} + a_{kF}) \\ &= a_{kB}a^{+}_{k'B} + a_{kB}a^{+}_{k'F} + a_{kF}a^{+}_{k'B} + a_{kF}a^{+}_{k'F} + a^{+}_{k'B}a_{kB} + a^{+}_{k'B}a_{kF} + a^{+}_{k'F}a_{kB} + a^{+}_{k'F}a_{kF} \\ &= 2a_{kB}a^{+}_{k'B} - \delta_{kk'} + (a_{kB}a^{+}_{k'F} + a^{+}_{k'F}a_{kB}) + (a_{kF}a^{+}_{k'B} + a^{+}_{k'B}a_{kF}) + (a_{kF}a^{+}_{k'F} + a^{+}_{k'F}a_{kF}) \end{aligned} \tag{3.10}$$

Now if we assume that the product of a boson operator and a fermion operator leads to an operator that should obey anti – commutation laws, then for $k = k'$,

$$a_{kB}a^{+}_{kF} + a^{+}_{kF}a_{kB} = 1 \tag{3.11}$$

This is physically acceptable since if a boson and a fermion came together to form a molecule, then the molecule is a fermion. Similarly, the other term in Eq. (3.10) becomes 1, thus,

$$\begin{aligned}\{\alpha_k, \alpha_k^+\} &= 2a_{kB}a_{kB}^+ - 1 + 1 + 1 + 1 = 2(a_{kB}a_{kB}^+ + 1) \\ &= 2(a_{kB}^+ a_{kB} + 2) = 2(n_{kB} + 2)\end{aligned} \tag{3.12}$$

If, however, we assume that the product of a boson operator and a fermion operator leads to an operator that should obey commutation laws, then,

$$a_{kB}a_{k'F}^+ - a_{k'F}^+ a_{kB} = \delta_{kk'} \tag{3.13a}$$

Or

$$a_{kB}a_{kF}^+ - a_{kF}^+ a_{kB} = 1 \text{ for } k = k' \tag{3.13b}$$

Corresponding to Eq. (3.13), i can also write,

$$a_{kF}a_{kB}^+ - a_{kB}^+ a_{kF} = 1 \tag{3.14}$$

Substituting from Eq. (3.13) and Eq. (3.14) in Eq. (3.10) for $k = k'$ yields,

$$\begin{aligned}\{\alpha_k, \alpha_k^+\} &= 2a_{kB}a_{kB}^+ - 1 + (2a_{kB}a_{kF}^+ - 1) + (2a_{kF}a_{kB}^+ - 1) + 1 \\ &= 2(a_{kB}a_{kB}^+ + a_{kB}a_{kF}^+ + a_{kF}a_{kB}^+ - 1)\end{aligned} \tag{3.15}$$

Equating Eq. (3.12) to Eq. (3.15) becomes,

$$a_{kB}a_{kF}^+ + a_{kF}a_{kB}^+ = 2 \tag{3.16}$$

Because of the positive sign between the two terms, it is an anti-commutator. This is the commutation law between the old operators for boson and fermion when we have a mixture of free bosons and fermions. The first term in Eq. (3.16) means that in the k^{th} level when a boson is destroyed, a fermion is created; and in the second term it means that in the k^{th} level when a fermion is destroyed, a boson is created. This leads to conservation of bosons and fermions in the k^{th} level, and thereby in the whole assembly.

From Eq. (3.08) it can be shown that,

$$\alpha_k a_{kF}^{+} = a_{kB} a_{kF}^{+} + a_{kF} a_{kF}^{+}$$
$$\alpha_k a_{kB}^{+} = a_{kB} a_{kB}^{+} + a_{kF} a_{kB}^{+} \tag{3.17}$$

which can be added to give,

$$\alpha_k (a_{kF}^{+} + a_{kB}^{+}) = (a_{kB} a_{kF}^{+} + a_{kF} a_{kB}^{+}) + (a_{kF} a_{kF}^{+} + a_{kB} a_{kB}^{+})$$
$$\alpha_k \alpha_k^{+} = +n_{kB} - n_{kF} + 4 \tag{3.18}$$

Similarly, using Eq. (3.11) and similar equations that exist in Eq. (3.10), it can be shown that,

$$\alpha_k^{+} (a_{kF} + a_{kB}) = (a_{kB}^{+} + a_{kF}^{+})(a_{kB} + a_{kF})$$

Or

$$\alpha_k^{+} \alpha_k = (a_{kB}^{+} a_{kB} + a_{kF}^{+} a_{kF}) + (a_{kB}^{+} a_{kF} + a_{kF}^{+} a_{kB})$$

$$\alpha_k^{+} \alpha_k = n_{kB} + n_{KF} \tag{3.19}$$

Using Eq. (3.08), Eq. (3.18) and Eq. (3.19), it can be shown that,

$$\alpha_k \alpha_k^{+} + \alpha_k^{+} \alpha_k = 2(n_{kB} + 2) = \{\alpha_k, \alpha_k^{+}\} \tag{3.20}$$

Eq. (3.12) and Eq. (3.20) are identically equal. This confirms the correctness of the algebraic scheme that we have developed so far.

Now suppose,

$$\begin{aligned}\alpha_k &= U_k a_{kB} a_{kF}^+ - V_k a_{kF} a_{kB}^+ \\ \alpha_k^+ &= U_k a_{kB}^+ - V_k a_{kB}^+ a_{kF}\end{aligned} \tag{3.21}$$

Substituting Eq. (3.15) in Eq. (3.21) yields,

$$= \left(U_k a_{kB} a_{kF}^+ - V_k a_{kF} a_{kB}^+\right)\left(U_k a_{kB}^+ - V_k a_{kB}^+ a_{kF}\right) + \left(U_k a_{kB}^+ - V_k a_{kB}^+ a_{kF}\right)\left(U_k a_{kB} a_{kF}^+ - V_k a_{kF} a_{kB}^+\right)$$

$$= \begin{pmatrix} U_k a_{kB} a_{kF}^+ U_k a_{kB}^+ - U_k a_{kB} a_{kF}^+ V_k a_{kB}^+ a_{kF} - U_k a_{kB}^+ V_k a_{kF} a_{kB}^+ + V_k a_{kB}^+ a_{kF} V_k a_{kF} a_{kB}^+ + U_k a_{kB}^+ U_k a_{kB} a_{kF}^+ \\ - U_k a_{kB}^+ V_k a_{kF} a_{kB}^+ - V_k a_{kB}^+ a_{kF} U_k a_{kB} a_{kF}^+ + V_k a_{kB}^+ a_{kF} V_k a_{kF} a_{kB}^+ \end{pmatrix}$$

$$\begin{aligned}\left\{\alpha_k, \alpha_k^+\right\} = \alpha_k \alpha_k^+ + \alpha_k^+ \alpha_k &= U_k^2 a_{kB} a_{kF}^+ a_{kB}^+ a_{kF} - U_k V_k a_{kB} a_{kF}^+ a_{kF}^+ a_{kB} \\ &- U_k V_k a_{kF} a_{kB}^+ a_{kB}^+ a_{kF} + V_k^2 a_{kF} a_{kB}^+ a_{kF}^+ a_{kB} \\ &+ U_k^2 a_{kB}^+ a_{kF} a_{kB} a_{kF}^+ - U_k V_k a_{kB}^+ a_{kF} a_{kF} a_{kB}^+ - U_k V_k a_{kF}^+ a_{kB} a_{kB} a_{kF}^+ \\ &+ V_k^2 a_{kF}^+ a_{kB} a_{kF} a_{kB}^+\end{aligned} \tag{3.22}$$

If we assume that a_{kB} and a_{kF}^+ anti-commute, and similarly we assume that a_{kF} and a_{kB}^+ anticommute then it implies that they obey the following anti-commution law,

$$\begin{aligned}\left\{a_{kB}, a_{kF}^+\right\} &= a_{kB} a_{kF}^+ + a_{kF}^+ a_{kB} = 0 \\ \left\{a_{kF}, a_{kB}^+\right\} &= a_{kF} a_{kB}^+ + a_{kB}^+ a_{kF} = 0\end{aligned} \tag{3.23}$$

Now evaluating the term in Eq. (3.22), and using Eq. (3.23) yields,

$$a_{kB} a_{kF}^+ a_{kB}^+ a_{kF} = -a_{kF}^+ a_{kB} (-a_{kF} a_{kB}^+) = a_{kF}^+ a_{kB} a_{kF} a_{kB}^+ \tag{3.24}$$

Similarly, the other terms in Eq. (3.22) can be re-written using Eq. (3.23). Using the terms like the one obtained in Eq. (3.24), Eq. (3.22) becomes,

$$\alpha_k \alpha_k^+ + \alpha_k^+ \alpha_k = \{\alpha_k , \alpha_k^+\} = 2(U_k^2 + V_k^2) - 2U_k V_k \qquad (3.25)$$

Now for the transformation represented by $\alpha_k^{,} s$ to be canonical, it must have

$$\{\alpha_k , \alpha_k^+\} = 1 = 2(U_k^2 + V_k^2) - 2U_k V_k = (U_k^2 + V_k^2) + (U_k - V_k)^2$$

Or

$$U_k^2 + V_k^2 = 1 - (U_k - V_k)^2 \qquad (3.26)$$

Or

$$(U_k^2 - V_k^2) = 2U_k^2 + (U_k - V_k)^2 - 1 \qquad (3.27)$$

$$(U_k^2 - V_k^2) = 1 - 2V_k^2 - (U_k - V_k)^2 \qquad (3.27a)$$

For $U_k = V_k = \frac{1}{\sqrt{2}}$ the left and the right-hand side of Eq. (3.26) and Eq. (3.27) are equal.

Other solutions could be $V_k = 0, U_k = \frac{1}{\sqrt{2}}$; and $U_k = 0, V_k = \frac{1}{\sqrt{2}}$, on using Eq. (3.26) and Eq. (3.27), the following values for U_k and V_k are obtained,

$$V_k = 0, U_k = \frac{1}{\sqrt{2}} \qquad (3.28)$$

$$U_k = 0, V_k = \frac{1}{\sqrt{2}} \qquad (3.29)$$

From Eqs. (3.28) and (3.29). Equations Eq. (3.21) the old operators in terms of the new operators $\alpha_k^{,} s$ can be solved to obtain,

$$a_{kF}a_{kB}^{+} = \frac{1}{(U_k^2 - V_k^2)}(V_k\alpha_k + U_k\alpha_k^{+}) \tag{3.30}$$

$$a_{kB}a_{kF}^{+} = \frac{1}{(U_k^2 - V_k^2)}(U_k\alpha_k + V_k\alpha_k^{+}) \tag{3.31}$$

From Eq. (3.30) and Eq. (3.31) it can written respectively as:

$$a_{kF}^{+}a_{kB} = \frac{1}{(U_k^2 - V_k^2)}(V_k\alpha_k^{+} + U_k\alpha_k) \tag{3.32}$$

$$a_{kB}^{+}a_{kF} = \frac{1}{(U_k^2 - V_k^2)}(U_k\alpha_k^{+} + V_k\alpha_k) \tag{3.33}$$

This will make it easy when writing combinations of a^{+} and a's that are part of the new operators α .

3.3 Quasi-Particle Energy, E_k , of a Mixture of Interacting Bosons and Fermions

In the recent past, mixtures of fermions in optical lattice (Bloch *et al*., 2008), mixtures of bosons in optical lattice Georges, 2007), and mixtures of interacting boson-fermion atoms (Anderson, *et. al*.,1995; DeMarco and Jin, 1999 and Bose, 1924) have been studied theoretically and experimentally to obtain stability conditions, and the conditions for demixing. In another experimental study, sympathetic cooling of bosonic and fermionic lithium gases was attempted to obtain conditions of quantum degeneracy (Schreck *et al*., 2001). Properties of a quasi-pure Bose-Einstein Condensate immersed in a fermi sea have also been studied (Nygaard, *et. al*., 1999; Ryder, 1996 and Blassome, *et. al*.,1995).

Zero -temperature phase diagram of a gas mixture composed of bosonic and fermionic atoms interacting through a Feshbach resonance have been studied experimentally and theoretically. For such systems, phase diagrams showing separate phases were obtained, and they showed both first order and second order phase transitions. It was found that close to unitarity (when the scattering length 'a' becomes infinity, i.e., $a \to \infty$), there is a regime in which there is a phase separation among the systems of the mixture (which means that the core may be bosons surrounded by the fermions, or the core may be fermions and it is surrounded by bosons). This situation is known as phase separation of fermions and bosons. It should be understood that in the low temperature limit, the only interaction is in the S-channel. In this limit, the fermions do not interact due to Pauli exclusion principle. The bosons interact repulsively among themselves such that their scattering length (a_{BB}); is positive and there may be attractive interaction between bosons and fermions with a negative scattering length ($-a_{FB}$). Such a situation can also be described by saying that if the energy of the bound state is made to cross the bottom of continuum i.e., the energy that the boson and fermion have when they are far apart and at rest), then the S-wave scattering length of the boson and fermion will diverge (means $a \to \infty$). Alternatively, when the system is in the bound state explicitly, the scattering lengths will be finite, and not diverge. There are a number of methods to obtain degenerate boson-fermion mixture in the limit of very low temperature. One of the methods is known as Feshbach resonance in which a bound state of a boson and fermion (boson -fermion) appears around zero energy. The bosons and fermions can interact by forming a molecule, heteronuclear boson-fermion mixtures were studied by Feshbach resonance and were used to study especially the properties of Bose -Fermi mixture such as ${}^{87}_{37}Rb{-}{}^{40}_{19}K$.

Another experimental setup for studying degenerate Fermi-Bose mixtures is the so-called optical lattice method. Such mixtures have been studied theoretically also. In this thesis a degenerate heteronuclear mixture of bosons and fermions has been studied theoretically at very low temperatures. Degenerate mixtures of a bosonic and fermionic atomic gas in an optical lattice have been studied successfully. Such degenerate mixtures at very low temperatures cooled down to nanokelvin, or sometimes to picokelvin temperatures, to facilitate the experimental study of quantum phase transition in systems of mixed quantum statistics.

A model Hamiltonian, H, for such an assembly is written in terms of the parameters of interactions involved, and it is then diagonalized by using the canonical transformation worked out in section 3.2 and given by Eq. (3.30)-Eq. (3.33). The resulting quasi-particle energy spectrum, E_k , is obtained in terms of parameters that define the degenerate boson -fermion mixture.

3.4 Model Hamiltonian H, and its Diagonalization

H was given by Eq.32 and can now be written as

$$= \sum \varepsilon_{kB} a^{+}_{kB} a_{kB} + \sum_{k} \varepsilon_{kF} a^{+}_{kF} a_{kF} + \frac{1}{2} \sum G_{BF} a^{+}_{k_1 B} a^{+}_{k_2 F} a_{k'_2 F} a_{k'_1 B} \tag{3.34}$$

In the last term, momentum conservation is assumed so that the summation is carried over all values of k_1, k_2, k'_1, k'_2 such that $k_1 + k_2 = k'_1 + k'_2$

In the above equation, a boson with momentum k_1' is destroyed and a boson with momentum k_1 is created or a particle with momentum k_1' is destroyed and goes to reappear as a particle with momentum k_1. The momentum transfer is $k_1 - k_1'$. Similarly, a fermion with momentum k_2' is destroyed and reappears as a fermion with momentum k_2. The momentum transfer is $k_2 - k_2'$.

For the conservation of momentum of these two particles, the momentum transfer for both of these must be equal in magnitude but opposite in sign, i.e.,

$$k_1 - k_1' = -(k_2 - k_2') = -k_2 + k_2' \quad \text{or} \quad k_1 + k_2 = k_1' + k_2' \tag{3.35}$$

In the boson – fermion interaction, a pair of boson –fermion may be destroyed, and a pair of boson – fermion may be created such that momentum is conserved; $k_1 + k_2 = k_1' + k_2'$.

Different combinations of k Values could be as follows:

$k_1 = k_2 = k_1' = k_2' = 0$.

$k_1 = k_1' = 0$ and $k_2 = k_2' = k$ or alternatively $k_2 = k_2' = 0$ and $k_1 = k_1' = k$.

These two possibilities will give a factor of 2 for identical particles

$k_1' = k_2 = 0$ and $k_1 = k_2' = k$ or alternatively $k_1 = k_2' = 0$ and $k_1' = k_2 = k$. These two possibilities will give a factor of 2 for identical particles

$k_1 = k_2 = 0$ and $k_1' = -k_2' = k$ or alternatively $k_1' = k_2' = 0$ and $k_1 = -k_2 = k$

Here, the invariance of the two - particle interaction under time reversal such that,

$$G_k = G_{-k} or G_{BF} = G_{-BF} \tag{3.36}$$

At a very low temperature, when the assembly of bosons and fermions is in the condensed phase, the Hamiltonian H can be written as

$$\begin{aligned}
H &= \sum_k \varepsilon_{kB} a_{kB}^+ a_{kB} + \sum_k \varepsilon_{kF} a_{kF}^+ a_{kF} + \frac{1}{2} G_{BF}^0 n_{0B} n_{0F} \\
&+ \frac{1}{2} G_{BF}^0 n_{0B} \sum_k a_{kF}^+ a_{kF} + \frac{1}{2} G_{BF}^0 n_{0F} \sum_k a_{kB}^+ a_{kB} \\
&+ \left(-\frac{1}{2}\right) \sum_k G_{BF}^k a^+{}_{kB} a_{kF} a_{0F}^+ a_{0B} - \frac{1}{2} \sum_k G_{BF}^{-k} a^+{}_{0B} a_{0F} a_{kF}^+ a_{kB} \\
&- \frac{1}{2} \sum_k G_{BF}^k a_{0B}^+ a_{-kF} a_{0F}^+ a_{kB} - \frac{1}{2} \sum_k G_{BF}^k a_{kB}^+ a_{0F} a_{-kF}^+ a_{0B}
\end{aligned} \tag{3.37}$$

In Eq. (3.37), the summation over k is for all values of k except $k = 0$ since the occupation of the $k = 0$ has been taken care of separately. To transform Eq. (3.37) into new operators $\alpha's$, we have to use Eq. (3.30) and Eq. (3.31) and Eq. (3.32) and Eq. (3.33). In fact, the last two terms in Eq. (3.37) can be dropped since these correspond to interaction between fermions trapped in the Fermi sea (k<0) and bosons at (k=0) or above (k=0). Such interactions are negligible thus Eq. (3.37) can be written as,

$$\begin{aligned}
H &= \sum_k \varepsilon_{kB} a_{kB}^+ a_{kB} + \sum_k \varepsilon_{kF} a_{kF}^+ a_{kF} + \frac{1}{2} G_{BF}^0 n_{0B} n_{0F} + \frac{1}{2} G_{BF}^0 n_{0B} \sum_k a_{kF}^+ a_{kF} \\
&+ \frac{1}{2} G_{BF}^0 n_{0F} \sum_k a_{kB}^+ a_{kB} - \frac{1}{2} \sum_k G_{BF}^k a^+{}_{kB} a_{kF} a_{0F}^+ a_{0B} - \frac{1}{2} \sum_k G_{BF}^{-k} a_{0B}^+ a_{0F} a_{kF}^+ a_{kB}
\end{aligned} \tag{3.38}$$

In Eq. (3.38) the first term corresponds to the energy of bosons, the second term corresponds to the energy of fermions. The third term corresponds to the interaction energy of the bosons and fermions in the state above $k = 0$, the fifth term corresponds to the interaction energy of the fermions in the ZMS and the bosons in the state above $k = 0$, the sixth and seventh terms correspond to the interaction energy between bosons and fermions in the ZMS and all the levels above $k = 0$.

We are now to transform each term in Eq. (3.38) into new operators α 's . For pure bosons (Khanna 1986),

$$a_{kB} = U_k \alpha_k + V_k \alpha^+_{-k} \qquad (3.39)$$

And for fermions,

$$a_{kF} = U_k \alpha_k + V_k \beta^+_k \qquad (3.40)$$

where $\beta^+_k = U_k a_k + V_k a^+_k$

We can replace α^+_{-k} by α^+_k since the particle in the states $k < 0$ do not contribute to the physics of the problem or it can be written as,

$$a_{kB} = U_k \alpha_k + V_k \alpha^+_k \qquad (3.41)$$

Values of a_{kB} and a_{kF} from Eq. (3.41) and Eq. (3.40) will be substituted in Eq. (3.38) to convert H into the new operators $+\alpha$'s. To diagonalize the Hamiltonian, H , the values have to be written term by term. Thus,

$$\begin{aligned} a^+_{kB} a_{kB} &= \left(U_k \alpha^+_k + V_k \alpha_k\right)\left(U_k \alpha_k + V_k \alpha^+_k\right) \\ &= U_k^2 \alpha^+_k \alpha_k + U_k V_k \alpha^+_k \alpha^+_k + V_k U_k \alpha_k \alpha_k + V_k^2 \alpha_k \alpha^+_k \\ &= U_k^2 \alpha^+_k \alpha_k + U_k V_k \alpha^+_k \alpha^+_k + V_k U_k \alpha_k \alpha_k + V_k^2 \left(1 + \alpha^+_k \alpha_k\right) \\ &= \left(U_k^2 + V_k^2\right) \alpha^+_k \alpha_k + V_k^2 + U_k V_k \alpha^+_k \alpha^+{}_k + V_k U_k \alpha_k \alpha_k \end{aligned} \qquad (3.42)$$

And ;

$$
\begin{aligned}
a_{kF}^{+}a_{kF} &= \left(U_k\alpha_k^{+} + V_k\beta_k\right)\left(U_k\alpha_k + V_k\beta_k^{+}\right) \\
&= U_k^2\alpha_k^{+}\alpha_k + U_kV_k\alpha_k^{+}\beta_k^{+} + V_kU_k\beta_k\alpha_k + V_k^2\beta_k\beta_k^{+} \\
&= U_k^2\alpha_k^{+}\alpha_k + U_kV_k\alpha_k^{+}\beta_k^{+} + V_kU_k\beta_k\alpha_k + V_k^2\left(1-\beta_k^{+}\beta_k\right) \\
&= U_k^2\alpha_k^{+}\alpha_k + U_kV_k\alpha_k^{+}\beta_k^{+} + V_kU_k\beta_k\alpha_k + V_k^2 - V_k^2\beta_k^{+}\beta_k \\
&= U_k^2\alpha_k^{+}\alpha_k - V_k^2\beta_k^{+}\beta_k + U_kV_k\left(\alpha_k^{+}\beta_k^{+} + \beta_k\alpha_k\right)
\end{aligned} \tag{3.43}
$$

And $a_{kB}^{+}a_{kF}a_{0F}^{+}a_{0B} = \left(U_k\alpha_k^{+} + V_k\alpha_k\right)\left(U_k\alpha_k + V_k\beta_k^{+}\right)\left(U_0\alpha_0^{+} + V_0\beta_0\right)\left(U_0\alpha_0 + V_0\alpha_0^{+}\right)$

$$= \left[\left(U_k\alpha_k^{+} + V_k\alpha_k\right)\left(U_k\alpha_k + V_k\beta_k^{+}\right)\right]\left[\left(U_0\alpha_0^{+} + V_0\beta_0\right)\left(U_0\alpha_0 + V_0\alpha_0^{+}\right)\right]$$

$$= \left[U_k^2\alpha_k^{+}\alpha_k + U_kV_k\alpha_k^{+}\beta_k^{+} + U_kV_k\alpha_k\alpha_k + V_k^2\alpha_k\beta_k^{+}\right]$$
$$\left[U_0^2\alpha_0^{+}\alpha_0 + U_0V_0\alpha_0^{+}\alpha_0 + U_0V_0\beta_0\alpha_0 + V_0^2\beta_0\alpha_0^{+}\right]$$

$$= U_k^2U_0^2\alpha_k^{+}\alpha_k\ \alpha^{+}{}_0\alpha_0 + U_k^2U_0V_0\alpha_k^{+}\alpha_k\alpha_0^{+}\alpha_0 + U_k^2U_0V_0\alpha_k^{+}\alpha_k\beta_0\alpha_0 + U_k^2V_0^2\alpha_k^{+}\alpha_k\beta_0\alpha_k^{+}$$

$$+U_kV_kU_0^2\alpha_k^{+}\beta_k^{+}\alpha_0^{+}\alpha_0 + U_kV_kU_0V_0\alpha_k^{+}\beta_k^{+}\alpha_0^{+}\alpha_0 + U_kV_kU_0V_0\alpha_k^{+}\beta_k^{+}\beta_0\alpha_0 + U_kV_kV_0^2\alpha_k^{+}\beta_k^{+}\beta_0\alpha_0^{+} +$$
$$U_kV_kU_0^2\alpha_k\alpha_k\alpha_0^{+}\alpha_0 + U_kV_kU_0V_0\alpha_k\alpha_k\alpha_0^{+}\alpha_0 + U_kV_kU_0V_0\alpha_k\alpha_k\beta_0\alpha_0 + U_kV_kV_0^2\alpha_k\alpha_k\beta_0\alpha_0^{+}$$

$$+V_k^2U_0^2\alpha_k\beta_k^{+}\alpha_0^{+}\alpha_0 + V_k^2U_0V_0\alpha_k\beta_k^{+}\alpha_0^{+}\alpha_0 + V_k^2U_0V_0\alpha_k\beta_k^{+}\beta_0\alpha_0 + V_k^2V_0^2\alpha_k\beta_k^{+}\beta_0\alpha_0^{+} \tag{3.44}$$

And

$$a_{0B}^{+}a_{0F}a_{kF}^{+}a_{kB} = \left(U_0\alpha_0^{+} + V_0\alpha_0\right)\left(U_0\alpha_0 + V_0\beta_0^{+}\right)\left(U_k\alpha_k^{+} + V_k\beta_k\right)\left(U_k\alpha_k + V_k\alpha_k^{+}\right)$$

$$\left[\left(U_0\alpha_0^{+} + V_0\alpha_0\right)\left(U_0\alpha_0 + V_0\beta_0^{+}\right)\right]\left[\left(U_k\alpha_k^{+} + V_k\beta_k\right)\left(U_k\alpha_k + V_k\alpha_k^{+}\right)\right]$$

$$
\begin{aligned}
&\left[U_0^2\alpha_0^{+}\alpha_0 + U_0V_0\alpha_0^{+}\beta_0^{+} + V_0U_0\alpha_0\alpha_0 + V_0^2\alpha_0\beta_0^{+}\right]\left[U_k^2\alpha_k^{+}\alpha_k + U_kV_k\alpha_k^{+}\alpha_k^{+} + V_kU_k\beta_k\alpha_k + V_k^2\beta_k\alpha_k^{+}\right] \\
&= U_0^2U_k^2\alpha_0^{+}\alpha_0\alpha_k^{+}\alpha_k + U_0^2U_kV_k\alpha_0^{+}\alpha_0\alpha_k^{+}\alpha_k^{+} + U_0^2U_k^2\alpha_0^{+}\alpha_0\alpha_k^{+}\alpha_k + U_0^2V_k^2\alpha_0^{+}\alpha_0\beta_k\alpha_k^{+} \\
&+U_0V_0U_k^2\alpha_0^{+}\beta_0^{+}\alpha_k^{+}\alpha_k + U_0V_0U_kV_k\alpha_0^{+}\beta_0^{+}\alpha_k^{+}\alpha_k^{+} + U_0V_0V_kU_k\alpha_0^{+}\beta_0^{+}\beta_k\alpha_k + U_0V_0V_k^2\alpha_0^{+}\beta_0^{+}\beta_k\alpha_k^{+} \\
&+V_0U_0U_k^2\alpha_0\alpha_0\alpha_k^{+}\alpha_k + V_0U_0U_kV_k\alpha_0\alpha_0\alpha_k^{+}\alpha_k^{+} + V_0U_0V_kU_k\alpha_0\alpha_0\beta_k\alpha_k + V_0U_0V_k^2\alpha_0\alpha_0\beta_k\alpha_k^{+} \\
&+V_0^2U_k^2\alpha_0\beta_0^{+}\alpha_k^{+}\alpha_k + V_0^2U_kV_k\alpha_0\beta_0^{+}\alpha_k^{+}\alpha_k^{+} + V_0^2V_kU_k\alpha_0\beta_0^{+}\beta_k\alpha_k + V_0^2V_k^2\alpha_0\beta_0^{+}\beta_k\alpha_k^{+}
\end{aligned} \tag{3.45}
$$

In Eq. (3.42), Eq. (3.43), Eq. (3.44) and Eq. (3.45), we shall retain only the diagonal terms and put the non-diagonal terms equal to zero. To diagonalize the Hamiltonian *H*, the following are the diagonal terms in Eq. (3.42) to Eq. (3.45). This leads to,

$$\left(U_k^2+V_k^2\right)\alpha_k^+\alpha_k+V_k^2 \tag{3.46}$$

From Eq. (3.43), Eq. (3.46) becomes,

$$U_k^2\alpha_k^+\alpha_k-V_k^2\beta_k^+\beta_k+V_k^2 \tag{3.47}$$

And from Eq. (3.44) it becomes,

$$U_k^2U_0^2\alpha^+{}_k\alpha_k\alpha_0^+\alpha_0\ \alpha^+{}_0\alpha_0+U_kV_kU_0V_0\alpha_k^+\beta_k^+\beta_0\alpha_0+U_kV_kU_0V_0\alpha_k\alpha_k\alpha_0^+\alpha_0+U_k^2V_0^2\alpha_k\beta_k^+\beta_0\alpha_0^+\ \ldots \tag{3.48}$$

Similarly, from Eq. (3.45) it also becomes,

$$U_0^2U_k^2\alpha_0^+\alpha_0\alpha_k^+\alpha_k+U_0V_0U_kV_k\alpha_0\alpha_0\alpha_k^+\alpha_k^++V_0^2V_k^2\alpha_0\beta_0^+\beta_k\alpha_k^+ \tag{3.49}$$

Now the diagonalized Hamiltonian can be written as,

$$\begin{aligned}
H &= \varepsilon_{kB}\left[\left(U_k^2+V_k^2\right)\alpha_k^+\alpha_k+V_k^2\right]+\sum_k\varepsilon_{kF}\left[U_k^2\alpha_k^+\alpha_k-V_k^2\beta_k^+\beta_k+V_k^2\right]+\frac{1}{2}G_{BF}^0\eta_{0B}\eta_{0F}\\
&+\frac{1}{2}G_{BF}^0\eta_{0B}\sum_k\left[U_k^2\alpha_k^+\alpha_k-V_k^2\beta_k^+\beta_k+V_k^2\right]+\frac{1}{2}G_{BF}^0\eta_{0F}\sum_k\left(U_k^2+V_k^2\right)\alpha_k^+\alpha_k+V_k^2\\
&+\frac{1}{2}\sum_k G_{BF}^k\left[U_k^2U_0^2\alpha_k^+\alpha_k\alpha_0^+\alpha_0+U_k^2U_0V_0\alpha_k^+\alpha_k\alpha_0^+\alpha_0+U_k^2U_0V_0\alpha_k^+\alpha_k\beta_0\alpha_0+U_k^2V_0^2\alpha_k^+\alpha_k\beta_0\alpha_0^+\right]\\
&+\left(-\frac{1}{2}\right)\sum_k G_{BK}^{-k}\left[U_0^2U_k^2\alpha_0^+\alpha_0\alpha_k^+\alpha_k+U_0V_0U_kV_k\alpha_0^+\beta_0^+\beta_k\alpha_k+U_0V_0U_kV_k\alpha_0\alpha_0\alpha_k^+\alpha_k^++U_0^2V_k^2\alpha_0\beta_0^+\beta_k\alpha_k^+\right]
\end{aligned} \tag{3.50}$$

Eq. (3.50) is obtained by subtracting the diagonalized terms from Eq. (3.46) to Eq. (3.49) in Eq. (3.38)

The non- diagonal terms of the Hamiltonian *H*, from Eq. (3.44) are,

$$\frac{1}{2}\sum_k G_{BF}^k \begin{bmatrix} U_k^2U_0V_0\alpha_k^+\alpha_k\alpha_0^+\alpha_0^+ + U_k^2U_0V_0\alpha_k^+\alpha_k\beta_0\alpha_0 + U_k^2V_0^2\alpha_k^+\alpha_k\beta_0\alpha_0^+ \\ +U_kV_kU_0^2\alpha_k^+\beta_k^+\alpha_0^+\alpha_0 + U_kV_kU_0V_0\alpha_k^+\beta_k^+\alpha_0^+\alpha_0 + U_kV_kV_0^2\alpha_k^+\beta_k^+\beta_0\alpha_0^+ \\ +U_kV_kU_0^2\alpha_k\alpha_k\alpha_0^+\alpha_0 + U_kV_kU_0V_0\alpha_k\alpha_k\beta_0\alpha_0 + U_kV_kV_0^2\alpha_k\alpha_k\beta_0\alpha_0^+ \\ +V_k^2U_0^2\alpha_k\beta_k^+\alpha_0^+\alpha_0 + V_k^2U_0V_0\alpha_k\beta_k^+\alpha_0^+\alpha_0^+ + V_k^2U_0V_0\alpha_k\beta_k^+\beta_0\alpha_0 \end{bmatrix} \quad (3.51)$$

And from Eq. (3.45) the non-diagonal terms are,

$$-\frac{1}{2}\sum_k G_{BF}^{-k} \begin{bmatrix} U_0^2U_kV_k\alpha_0^+\alpha_0\alpha_k^+\alpha_k^+ + U_0^2V_k^2\alpha_0^+\alpha_0\beta_k\alpha_k^+ \\ +U_0V_0U_k^2\alpha_0^+\beta_0^+\alpha_k^+\alpha_k + U_0V_0U_kV_k\alpha_0^+\beta_0^+\alpha_k^+\alpha_k^+ + U_0V_0V_k^2\alpha_0^+\beta_0^+\beta_k\alpha_k^+ \\ +V_0U_0U_k^2\alpha_0\alpha_0\alpha_k^+\alpha_k + V_0U_0V_kU_k\alpha_0\alpha_0\beta_k\alpha_k + V_0U_0V_k^2\alpha_0\alpha_0\beta_k\alpha_k^+ \\ +V_0^2U_k^2\alpha_0\beta_0^+\alpha_k^+\alpha_k + V_0^2U_kV_k\alpha_0\beta_0^+\alpha_k^+\alpha_k^+ + V_0^2V_kU_k\alpha_0\beta_0^+\beta_k\alpha_k \end{bmatrix} \quad (3.52)$$

The sum of non-diagonal terms given by Eq. (3.51) and Eq. (3.52) is to be put equal to zero. This term is to be denoted by,

$$H_2 = 0 \quad (3.53)$$

The energy Eigen-values for the assembly in the limit $T \to 0K$ are obtained from Eq. (3.50). Eq. (3.50) can now be written as,

$$H = \sum_k\left(\varepsilon_{kB} + \frac{1}{2}G_{BF}^0 n_{0F}\right)\left[\left(U_k^2 + V_k^2\right)\alpha_k^+\alpha_k + V_k^2\right] + \sum_k\left(\varepsilon_{kF} + \frac{1}{2}G_{BF}^0 n_{0B}\right)$$
$$\left[\left\{U_k^2\alpha_k^+\alpha_k - V_k^2\beta_k^+\beta_k + V_k^2\right\}\right] + \frac{1}{2}G_{BF}^0 n_{0B}n_{0F} \quad (3.54)$$

The energy – Eigen value of the Hamiltonian H in Eq. (3.54) are,

$$E = \frac{1}{2}G_{BF}^{0}n_{0B}n_{0F} + \sum_{k}\left(\varepsilon_{kB} + \frac{1}{2}G_{BF}^{0}n_{0F}\right)V_k^2 + \sum_{k}\left(\varepsilon_{kB} + \frac{1}{2}G_{BF}^{0}n_{0F}\right)\left(U_k^2 + V_k^2\right)$$

$$+ \sum_{k}\left(\varepsilon_{kF} + \frac{1}{2}G_{BF}^{0}n_{0B}\right)V_k^2 + \sum_{k}\left(\varepsilon_{kF} + \frac{1}{2}G_{BF}^{0}n_{0B}\right)\left(U_k^2 - V_k^2\right) \tag{3.54}$$

$$= \frac{1}{2}G_{BF}^{0}n_{0B}n_{0F} + \sum_{k}\left(\varepsilon_{kB} + \frac{1}{2}G_{BF}^{0}n_{0F}\right)\left(U_k^2 + 2V_k^2\right) + \sum_{k}\left(\varepsilon_{kF} + \frac{1}{2}G_{BF}^{0}n_{0B}U_k^2\right)$$

$$= \frac{1}{2}G_{BF}^{0}n_{0B}n_{0F} + \sum_{k}\left(\varepsilon_{kB} + \frac{1}{2}G_{BF}^{0}n_{0F}\right)2V_k^2 + \sum_{k}\left(\varepsilon_{kB} + \frac{1}{2}G_{BF}^{0}n_{0B} + \frac{1}{2}G_{BF}^{0}n_{0F}\right)U_k^2 \tag{3.56}$$

The bosonic contribution to the energy density ε_{kB} is given by,

$$\varepsilon_{kB} = \frac{\hbar^2}{2m_B}\left|\nabla\sqrt{n_B(x)}\right|^2 + U_B(x)n_B(x)\hbar^2 + \frac{2\pi a_{B0}\hbar^2}{m_B}n_B^2(x) \tag{3.57}$$

The fermionic contribution to the energy density ε_{kF} is given by (Roth *et al*., 2001, Nygaard *et al*., 1999),

$$\varepsilon_{kF} = \frac{3\left(6\pi^2\right)^{2/3}\hbar^2}{10m_F}n_F^{5/3} + U_F n_F\hbar^2 + \frac{\left(6\pi^2\right)^{5/3}a^3{}_{FI}\hbar^2}{5\pi m_F}n_F^{8/3} \tag{3.58}$$

where the first term represents the kinetic energy, the second term is the external trapping potential, and the third term is the p-wave fermion-fermion interaction. Eq. (3.58) is obtained by using the Thomas-Fermi approximation. Similarly, the Thomas – Fermi approximation can be used to calculate the contribution of the boson–fermion interaction to the energy density such that,

$$G_{BF} = \varepsilon_{BF} = \frac{4\pi a_{BFO}\hbar^2}{m_{BF}} n_B n_F + \frac{(6\pi)^{5/3} a^3{}_{BFI}\hbar^2}{10\pi m_{BF}} n_B n_F{}^{5/F} \tag{3.59}$$

where the first term describes the s-wave and the second term the p-wave boson-fermion interaction. For fermion number $\eta_F \approx 1000$, it is well known that Thomas-Fermi approximation is in good agreement with Hartree-Fock type calculation (Star *et al.*,2004). Here a_{BFO} is the interaction between bosons. Substituting from Eq. (3.56) to Eq. (3.59) in Eq. (3.55) yields,

$$E = \frac{1}{2} n_{0B} n_{0F} \left[\frac{4\pi a_{BFO} \mathrm{h}^2}{m_{BF}} n_B n_F + \frac{(6\pi^2)^{5/3} a^3{}_{BFI} \mathrm{h}^2}{10\pi m_{BF}} n_B n_F{}^{5/3} \right] +$$
$$\left\{ \begin{array}{l} \left[\frac{\mathrm{h}^2}{2m_B} \left| \nabla \sqrt{\eta_B(x)} \right|^2 + U_B(x) n_B(x) \mathrm{h}^2 + \frac{2\pi a_{B0} \mathrm{h}^2}{m_B} n_B{}^2(x) \right] + \\ \frac{n_{0F}}{2} \left[\frac{4\pi a_{BFO} \mathrm{h}^2}{m_{BF}} n_B n_F + \frac{(6\pi^2)^{5/3} a^3_{BF1} \mathrm{h}^2}{10\pi m_{BF}} n_B n_F{}^{5/3} \right] \end{array} \right\} 2V_k{}^2$$
$$\left\{ \left[\frac{3(6\pi^2)^{2/3} \mathrm{h}^2}{10 m_F} n_F^{5/3} + U_F n_{_F} \mathrm{h}^2 + \frac{(6\pi^2)^{5/3} a^3{}_{FI} \mathrm{h}^2}{5\pi m_F} n_F{}^{8/3} \right] + \frac{1}{2}(n_{0B} + n_{0F}) \left[\frac{4\pi a_{BFO} \mathrm{h}^2}{m_{BF}} n_B n_F + \frac{(6\pi^2)^{5/3} a^3{}_{BFI} \mathrm{h}^2}{10\pi m_{BF}} n_B n_F^{5/3} \right] \right\} U_k^2 \tag{3.60}$$

Eq. (3.60) represents the energy of the system in the ground state or ZMS.

Thus, $n_B = n_F$ will refer to the boson and fermion numbers in the ZMS, i.e. $n_B = n_{0B}$ and $n_F = n_{0F}$; U_B is trapping potential for boson and U_F is the trapping potential for fermion. For simplicity the trapping potentials U_B and U_F will be assumed to be the same if we restrict ourselves to parabolic trapping potentials with spherical symmetry, it can written that $U = \frac{1}{2} m\omega^2 \ell^2$, where ω is the oscillatory frequency and ℓ oscillator length $= (m\omega)^{-\frac{1}{2}}$.

$$U_F = U_B = \frac{\hbar^2}{2m\ell^2} \tag{3.61}$$

Where m is the mass of the particle and ℓ is the corresponding oscillatory length for bosons and fermions, assumed to be the same for boson and fermions. There are two sets of values,

$$U_k = V_k = \frac{1}{\sqrt{2}} \tag{3.62}$$

And the second set is

$$V_k = 0, U_k = \frac{1}{\sqrt{2}}$$
$$U_k = 0, V_k = \frac{1}{\sqrt{2}} \tag{3.63}$$

Using Eq. (3.61) and Eq. (3.62) in Eq. (3.60) leads to,

$$E = \frac{1}{2} n_{0B} n_{0F} \left[\frac{4\pi a_{BFO}\hbar^2}{m_{BF}} n_B n_F + \frac{(6\pi)^{5/3} a^3{}_{BFI}\hbar^2}{10\pi m_{BF}} n_B n_F^{5/3} \right] +$$
$$\left\{ \begin{array}{l} \left[\frac{1}{2m_B} \left| \nabla \sqrt{\eta_B(x)} \right|^2 + \frac{\hbar^2}{2m_B \ell^2} n_B(x) + \frac{2\pi a_{B0}\hbar^2}{m_B} n_B{}^2(x) \right] \\ + \frac{n_{0F}}{2} \left[\frac{4\pi a_{BFO}\hbar^2}{m_{BF}} n_B n_F + \frac{(6\pi^2)^{5/3} a^3{}_{BFI}\hbar^2}{10\pi m_{BF}} n_B n_F^{5/3} \right] \end{array} \right\}$$
$$\frac{1}{2} \left\{ \begin{array}{l} \left[\frac{3(6\pi^2)^{2/3}\hbar^2}{10 m_F} n_F^{5/3} + \frac{\hbar^2}{2m_F \ell^2} n_F(x) + \frac{(6\pi^2)^{5/3} a^3{}_{FI}\hbar^2}{5\pi m_F} n_F^{8/3} \right] \\ + \frac{1}{2}(n_{0B} + n_{0F}) \left[\frac{4\pi a_{BFO}\hbar^2}{m_{BF}} n_B n_F + \frac{(6\pi^2)^{5/3} a^3{}_{BFI}\hbar^2}{10\pi m_{BF}} n_B n_F^{5/3} \right] \end{array} \right\} \tag{3.64}$$

Using Eq. (3.61) and Eq. (3.63) in Eq. (3.60) results into two values of E, i.e.,

$$E = \frac{1}{2} n_{0B} n_{0F} \left[\frac{4\pi a_{BFO} \hbar^2}{m_{BF}} n_B n_F + \frac{(6\pi)^{5/3} a^3{}_{BFI} \hbar^2}{10\pi m_{BF}} n_B n_F^{5/3} \right]$$
$$+ \frac{1}{2} \{ \left[\frac{3(6\pi^2)^{2/3} \hbar^2}{10 m_F} \eta_F^{5/3} + \frac{\hbar^2}{2 m_F \ell^2} n_F(x) + \frac{(6\pi^2)^{5/3} a^3{}_{FI} \hbar^2}{10\pi m_{BF}} n_F^{8/3} \right]$$
$$+ \frac{1}{2} (n_{0B} + n_{0F}) \left[\frac{4\pi a_{BFO} \hbar^2}{m_{BF}} n_B n_F + \frac{(6\pi^2)^{5/3} a^3{}_{BFI} \hbar^2}{10\pi m_{BF}} n_B n_F^{5/3} \right] \}$$

(3.65)

and
$$E = \frac{1}{2} n_{0B} n_{0F} \left[\frac{4\pi a_{BFO} \hbar^2}{m_{BF}} n_B n_F + \frac{(6\pi)^{5/3} a^3{}_{BFI} \hbar^2}{10\pi m_{BF}} n_B n_F^{5/3} \right] + \left\{ \begin{array}{l} \left[\frac{\hbar^2}{2 m_B} \left| \nabla \sqrt{n_B(x)} \right|^2 + \frac{\hbar^2}{2 m_B l^2} n_B(x) + \frac{2\pi a_{B0} \hbar^2}{m_B} n_B^2(x) \right] + \\ \frac{\eta_{0F}}{2} \left[\frac{4\pi a_{BFO} \hbar^2}{m_{BF}} n_B n_F + \frac{(6\pi^2)^{5/3} a^3{}_{BFI} \hbar^2}{10\pi m_{BF}} n_B n_F^{5/3} \right] \end{array} \right\} \quad (3.66)$$

For $a_{BFI} \to 0,$ and for fixed density in the trap when $\nabla\sqrt{n(x)} = 0$, Eq.(3.66) becomes,

$$E = \frac{1}{2} n_{0B} n_{0F} \left[\frac{4\pi a_{BFO} \hbar^2}{m_{BF}} n_B n_F \right] + \frac{n_B \hbar^2}{2 m_B l^2} + \frac{2\pi a_{BO} \hbar^2 n_B^2}{m_B} + \frac{1}{2} n_{OF} \left[\frac{4\pi a_{BFO} \hbar^2}{m_{BF}} .n_B n_F \right]$$
$$+ \frac{n_F \hbar^2}{2 m_F l^2} + \frac{1}{2} n_{OB} \left[\frac{4\pi a_{BFO} \hbar^2}{m_{BF}} n_B n_F \right] \quad (3.67)$$

Where $n_B =$ number density of bosons in the mixture, $n_F =$ number density of fermions in the mixture, $n_{OB} =$ number density of bosons in the ZM, $n_{OF} =$ number density of fermions in the ZMS (below the Fermi surface), $a_{BO} =$ Boson-Boson, for S-waves., $a_{BF0} =$ Boson-Fermion, for S-waves.

In Eq.(3.67), the first term refers to a homogeneous gas of interacting bosons and fermions. The second term is the trapping potential for the bosons. The third term is due to the boson -boson interaction via the S-wave scattering. The fourth and the sixth terms are due to the S-wave scattering between bosons and fermions. The fifth term is the trapping potential for the fermions. Term four refers to the isolated fermions (n_{OF}) in the ZMS, and the term six refers to the isolated bosons (n_{OB}) in the ZMS. Thus the existence of term four will demand that $n_F > n_B$, and the existence of term six will demand that $n_B > n_F$. Term four can mean demixing of interacting boson-fermion system from the isolated fermions in ZMS, and the term six can likewise mean $m = m_B = m_F$ demixing of interacting boson-fermion system from the isolated bosons in the ZMS.

As a first approximation, values of different parameters could be assumed and the oscillator length of the trapping potentials $\ell = \ell_B = \ell_F$. Similarly the values of bosons and fermions particles are $n_B = n_F = 10^4 = n$. Assuming attractive boson – fermion interaction, and for a typical trap width length $\ell = 1\mu m$, a_{BF} is assumed to be, $a_{BF} = -50nm$. Another assumption could be made by p – wave scattering lengths. We can assume them to be very small $(a_{BFI} \to 0)$ and hence the corresponding p –wave term can be neglected in Eq.(3.64), Eq.(3.65) and Eq.(3.66) to calculate the value of *E*. With the above assumptions, Eq. (3.64) becomes,

$$E = \frac{1}{2} n^2 \left(\frac{4\pi a_{BFO} \hbar^2}{m} n^2 \right) + \left\{ \frac{\left| \nabla \sqrt{\eta(x)} \right|^2}{2m} + \frac{\hbar^2 n}{2m\ell^2} + \frac{2\pi a_{BO} \hbar^2}{m} n^2 + \frac{n}{2} \left(\frac{4\pi a_{BFO} \hbar^2}{m} n^2 \right) \right\} \quad (3.68)$$

And similarly Eq. (3.66) becomes,

$$E = \frac{1}{2} n^2 \left(\frac{4\pi a_{BFO} \hbar^2}{m} n^2 \right) + \frac{1}{2} \left\{ \frac{3\left(6\pi^2\right)^{2/3} \hbar^2}{10m} n^{5/3} + \frac{\eta \hbar^2}{2m\ell^2} + \frac{1}{2}\left(2n\right)\left[\frac{4\pi a_{BFO} \hbar^2}{m} n^2 \right] \right\} \tag{3.69}$$

And Eq. (4.64) becomes,

$$E = \frac{1}{2} n^2 \left(\frac{4\pi a_{BFO} \hbar^2}{m} n^2 \right) + \left\{ \frac{\left| \nabla \sqrt{n(x)} \right|}{2m} + \frac{\hbar^2 n}{2m\ell^2} + \frac{2\pi a_{BO} \hbar^2}{m} n^2 + \frac{n}{2}\left(\frac{4\pi a_{BFO} \hbar^2}{m} n^2 \right) \right\} \tag{3.70}$$

It is interesting to note that Eq. (3.68) and Eq. (3.69) are identical while Eq. (3.70) differs from both of them. Eq. (3.68) corresponds to $U_k = V_k = \frac{1}{\sqrt{2}}$, and Eq. (3.70) correspond to $U_k = 0$ and $V_k = \frac{1}{\sqrt{2}}$; whereas Eq.(3.69) correspond to $V_k = 0$ and $U_k = \frac{1}{\sqrt{2}}$.

Assuming the particle number density is fixed in the trap, and replacing $\nabla\sqrt{\eta(x)} = 0$, Eq.(3.68) and Eq.(3.70) then gives,

$$E = \frac{1}{2} n^2 \left(\frac{4\pi a_{BFO} \hbar^2}{m} n^2 \right) + \frac{n\hbar^2}{2m\ell^2} + \frac{2\pi a_{BO} \hbar^2}{m} n^2 + \frac{n}{2}\left(\frac{4\pi a_{BFO} \hbar^2}{m} n^2 \right) \tag{3.71}$$

The energy E is a polynomial in n since it has terms of the type n, n^2, n^3, n^4.

The expression for energy E in Eq. (3.71) contains the effect of Bose-Bose (BB) particle interaction and Bose –fermion particle interaction (BF), whereas Eq. (3.69) contains apparently the effects of BF interaction only.

The expression for quasi-particle energy E of a trapped and interacting assembly of bosons and fermions has been derived Eq. (3.71). There are two important terms in this energy expression. One corresponds to the boson-fermion condensate surrounded by fermions, and the other corresponds to the boson-fermion condensate surrounded by bosons. So far there is no any quasi particle energy expression in which this kind of segregation of condensates may have appeared after the canonical transformation that leads to quasi-particle energy expression.

The first term in Eq. (3.71) is comparatively the largest term, and hence the quasi-particle energy of an assembly of interacting bosons and fermions in the lowest (superfluid state) state can be approximately written as,

$$E = \frac{1}{2}\left(\frac{4\pi\hbar^2 a_{BFO}}{m}\right)n^4 \tag{3.72}$$

Eq.(3.72) is used to calculate E for a number of different boson-fermion mixtures studied so far experimentally since the experimental values of a_{BFO} will be needed in Eq.(3.72) to calculate E. Eq.(3.72) shows that the value of the quasi-particle energy depends on the boson-fermion scattering length a_{BFO}, the density of number of particles n, and the reduced mass m of the mixture. In fact, $n^4 = (n_F)^2 (n_B)^2$ where n_F and n_B are different.

CHAPTER FOUR

RESULTS AND DISCUSSIONS

4.1 Introduction

In this chapter, the results are presented and discussed.

4.2 Derivation of canonical transformation

In this thesis, a model of Hamiltonian that contains the energy for free interacting bosons, free interacting Fermions and further the interaction between fermions and bosons is developed and represented by Eq. (3.3) and Eq. (3.33). It is further diagonalized to arrive at Eq. (3.72) which is used to calculate the quasi-particle energy of the mixture.

4.3 Calculation of quasi-particle energy spectrum

The expression for E_k contains various parameters like the scattering length a_{BB} and a_{BF} , that determines the properties of such a system of combination of fermions and bosons. The values of E_k have been calculated for various combinations of boson and fermion mixtures using the experimental values for a_{BB} and a_{BF} for the corresponding combinations.

Eq. (3.72) is used to calculate the quasi-particle energy of a mixture of bosons and fermions. For a mixture composed of ${}^{7}_{3}Li$ (boson) and ${}^{6}_{3}Li$ (fermion),

$$a_{BFO} = 2.16nm = 21.6\dot{A} = 21.6\times10^{-8}cm \quad n = 10^4 \,; m = 10^{-24} g$$

The value of E will be,

$$E = \frac{1}{2}\left(\frac{4\pi\times10^{-54}\times21.6\times10^{-8}}{10^{-24}}\right)\times10^{16}$$

$$= 10^{-22} \text{ erg.}$$

The results show that, ultimately, the phase separation depends on the sign and magnitudes of the scattering length a_{BF} and a_{BB}. This kind of separation and demixing appears in Eq.(4.64b), and it is a consequence of canonical transformation developed in this thesis and then used for diagonalization to obtain the quasi-particle energy E of the boson-fermion mixture. Now for the different mixtures the values for η, ℓ, m, a_{BFO} and a_{BO} were selected from past experimental work.

In another set of experiments (Roth, 2012), the following data was obtained for different boson -fermion mixtures. For ${}^7_3Li - {}^6_3Li$, a_{BF} is positive ${}^7_3Li(Boson) - {}^6_3Li(fermion)$

First set

$a_B = -1.46nm; a_{BF} = +2.16nm$ attractive $a_B < 0(-ve)$ and $a_{BF} > 0(+ve)$ repulsion

$l_B = 1\mu m; \quad N_B = 10^4; N_F = 2.5\times10^4$

Second set

$a_B = 0.27nm; a_{BF} = 2.01nm$ repulsive, $l_B \approx 0.15\mu m$ (reduce the overlap of the two species)

Third set

$^{87}_{37}Rb\text{–}^{40}_{19}K$, a_{BF} 5.25nm (Repulsive) ; $a_{BF} = -13.8nm$ (Attractive)

Table 5.1 below shows the results of calculations of quasi-particle energy E_k got by taking different combinations of isotopes.

Table 5.1: Results of quasi-particle energy for the mixture of Bosons-Fermions

Table 5.1 shows that the value of E_k increases as the scattering length a_{BF} increases for the mixture $^{87}_{37}Rb + ^{40}_{19}K$, and it goes to negative when a_{BF} negative or smaller. Large a_{BF} means that the interaction is spread over a longer distance and hence the energy is comparatively large. This means it can sustain larger velocity of flow to sustain the superfluid state.

BOSON -FERMIONS MIXTU RE	SCATTERING LENGTH $(a_{BF} \times 10^{-8} cm)$	QUASI-PARTICLE ENERGY in ergs
$^{87}_{37}Rb + ^{40}_{19}K$	150	1.2375×10^{-13}
	162	1.3365×10^{-13}
	300	2.475×10^{-13}
	209	-1.103×10^{-17}
$^{6}_{3}Li + ^{7}_{3}Li$	0.2158	5.227×10^{-17}
$^{23}_{11}Na + ^{6}_{3}Li$	-1.45	-1.22×10^{-14}

For $^{7}_{3}Li$ (boson) - $^{6}_{3}Li$ (fermion) mixture, the following Parameters may be used (Cornell and Wieman, 2002*a*):

$a_{BO} = -1.46nm$, $a_{BFO} = 2.16nm$, $\eta \approx 10^4$ particles, $\ell = 2.67\mu m$ given that

$1nm = 10^{-9}m$ and $1\mu m = 10^{-6}m$

Reduced mass is calculated using this formula $m_{BF} = \dfrac{m_B \times m_F}{m_B + m_F} = (1.66 \times 10^{-27})kg$.

The unit of energy $= \dfrac{1}{2 \times 1.64} \times 10^{39}$.

Thus energy $1.877 \times 10^{35} J$ is $\left(\dfrac{1.877 \times 10^{35}}{\dfrac{1}{2 \times 1.64} \times 10^{39}} \right)$ units of energy.

$\approx 2 \times 1.64 \times 1.877 \times 10^{-4}$ units of energy.

≈ 6.15656 units of energy.

Another set of values are

$a_{BO} = 0.27nm$ and $a_{BFO} = 2.01nm\ a$

The value of energy is found to be $E = 1.859 \times 10^{35} J$, when 1.859 units of energy involved .

Similarly for a mixture of ${}^{87}Rb(boson)$ and ${}^{40}K(\,fermion)$ the parameters are

$a_{BO} = 5.25nm$ and $a_{BFO} = -13.8nm$.

The value of energy is found to be $E = 1.433 \times 10^{35} J$ (Deh *et al*., 2002).

For $a_{BO} = 523.71nm$, $a_{BFO} = -405.9$ and $+405.9nm$, the value of energy is found to be $E = -3.984\times10^{35}\,J$ and $E = 7.235\times10^{35}\,J$ respectively.

Number of particles $n = 2000$ for both ^{87}Rb(Boson) and ^{40}K (fermion) , the mass is

$m_B = 1.45\times10^{-23}\,kg$ and $\dfrac{m_F}{m_B} = 0.463$ the value of energy is found to be $E = 7.3\times10^{35}\,J$.

In most of the theoretical or experimental investigation done in the recent past, the inter play between boson-fermion and boson-boson interaction (Anderson *et al*., 1995; Batroni *et al*., 1990; Bloch *et al*., 2008). Studies have shown that the boson -fermion attraction $(a_{BF} < 0)$ and boson -fermion repulsion $(a_{BF} > 0)$ can lead to spatial separation of bosons and fermions

In such a case, either boson occupy the central region of the trap (boson core) and the fermions constitute the shell around it, or fermions occupy the central region of the trap and bosons constitute the shell round it. It is the Bose-Fermi S-wave scattering lengths, with minus or plus sign, that determines the demixing and the stability of the boson-fermion mixture.

In the quasi-particle energy expression derived in this thesis, there are no terms that show demixing or segregation of bosons and fermions. There are the terms that show that there always exist a core made of interacting bosons and fermions, and under certain conditions this core may be surrounded by bosons when $n_B > n_F$, and it may be surrounded by fermions when $n_F > n_B$. It is therefore, reasonable to emphasize that this kind of result is a consequence of canonical transformation developed in this thesis.

In most of the studies in the recent past, the stability properties of trapped Bose-Fermi gases mixture have been studied by trapping gases in an isotropic potential at ultra-cold temperatures. The stability properties are strongly influenced by interactions between the bosons and fermions, and between bosons. Fermion -fermion interaction is ineffective at very low temperature due to suppression of the S-wave scattering between identical fermions. Studies have shown that the spatial distribution of a Bose -Fermi gas mixture at $T \to 0$ depends strongly on the relative sign and magnitude of the bose-fermion and bose-bose scattering length a_{BF} and a_{BB} (Molmer, 1998, and Plenio, *et. al*.,1999). They found the region of temperature range at which the phase separation of the mixture happens. Also the span of the region depends on the ratio of coupling constant, G_{BF} and G_{BB} or a_{BF} and a_{BB} . Fermion constitutes a core within the core condensate; $(G_{BF} / G_{BB} < \frac{m_F \omega_F^2}{m_B \omega_B^2})$, and for $(G_{BF} / G_{BB} > \frac{m_F \omega_F^2}{m_B \omega_B^2})$, fermions are repelled from the center of the trap and localize near the edge of the Bose condensate, i.e. phase separation occurs in the system. Hence phase separation depends mainly on the ratio of the two coupling constants, whose values are,

$$G_{BB} = \frac{4\pi\hbar^2}{m_B}.a_{BB} \tag{4.1}$$

$$G_{BF} = \frac{4\pi\hbar^2}{m_{BF}}.a_{BF} \tag{4.2}$$

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

Theoretical studies on the properties of boson-fermion gas mixtures were undertaken by developing canonical transformation in terms of canonical operators for bosons and fermions. The canonical transformation equations were derived and presented in equations (3.30) to (3.33) in chapter three of this thesis. These transformation equations were used in the model Hamiltonian and further diagonalized for the mixture of Boson and Fermions.

The derived canonical transformation was applied in the model Hamiltonian to calculate the quasi-particle excitation spectrum energy, E_k, for selected boson-fermion mixtures. For ${}^{87}_{37}Rb + {}^{40}_{19}K$ mixture with α_{BF} of $150 \times 10^{-8}\ cm$, $162 \times 10^{-8}\ cm$, $300 \times 10^{-8}\ cm$, and $-209 \times 10^{-8}\ cm$, the corresponding calculated values of E_k were $1.237 \times 10^{-12}\ ergs$, $1.337 \times 10^{-12}\ ergs$, $2.475 \times 10^{-12}\ ergs$, and $-1.103 \times 10^{-12}\ ergs$. Similarly, for ${}^{6}_{3}Li + {}^{7}_{3}Li$ mixture with α_{BF} of $0.2158 \times 10^{-8}\ cm$ had E_k of $5.227 \times 10^{-17}\ ergs$, while for ${}^{23}_{11}Na + {}^{6}_{3}Li$ mixture with α_{BF} of $-1.45 \times 10^{-8}\ cm$ had E_k of $-1.22 \times 10^{-21}\ ergs$. The negative values of E_k mean that the interaction is attractive, where interacting species overlap, whereas positive values of E_k mean that the interaction is repulsive, where the overlap of interacting species reduces.

6.2 Recommendations

i. Using the numerical values of quasi-particle energy E (Eq.3.72), calculations can also be done to calculate the Landau's critical velocity, C, that determines the upper limit on the velocity of flow of the superfluid such that if the velocity of flow $V > C$, the superfluid state is destroyed and experiments can be designed to measure this velocity.

ii. It will also be appropriate to study the conditions that may lead to demixing of the components of the mixture, and destruction of Superfluidity

iii. Stability of Boson - fermion mixture has been studied on the basis of the number of bosons and fermions and in terms of the sign of inter particle interaction whether the a_{BF} is attractive and or repulsive (Karpiuk and Brewczyk, 2005). Consequently,

iv. More studies need to be done as to how the density of species, inter-particle interaction, and the size and depth of trap can determine stability of the mixture.

v. In the limit of very low temperatures ($T \to 0K$), the mixture will be in the superfluid state. The Landau criteria on the velocity of flow can be used to study the conditions that may be satisfied for the mixture to be in the super-fluid state. It is necessary to study both experimentally and theoretically whether the positive scattering length $(a_{BF} > 0)$ that corresponds to repulsive interaction will lead to a more stable superfluid state, or $(a_{BF} < 0)$ that refers to negative scattering length and hence attractive interaction.

vi. Another possible physical state could be when $N_B > N_F$ and when $N_F > N_B$. At very low temperatures, and $N_B > N_F$, bosons in the mixture can be assumed to be in a pure Bose-Einstein Condensate surrounded by fermions . The fermions may interact harmonically with the boson - boson pair, and the interaction may be a perturbation. Similarly for $N_F > N_B$, the bosons may surround the fermion-fermion system, and act as a harmonic perturbation. Using Feshbach resonance method The stability of such systems could be studied experimentally and theoretically.

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APPENDICES

APPENDIX I: SIMILARITY REPORT

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