THE CORRELATION BETWEEN NUCLEON-NUCLEON INTERACTION, PAIRING ENERGY GAP AND PHASE SHIFT FOR IDENTICAL NUCLEONS IN NUCLEAR SYSTEMS.

BY

WILLY KIPRONO KOECH

A THESIS SUBMITTED TO THE SCHOOL OF SCIENCE IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE IN PHYSICS, UNIVERSITY OF ELDORET, KENYA

MAY, 2021

DECLARATION

Declaration by the Candidate

This thesis is my original work and it has not been presented in any other university for consideration of any certification. No part of this thesis may be reproduced without the prior written permission of the author and/or University of Eldoret.

…………………………..…… Date …………………………

Koech Kiprono Willy

SC/PGP/003/2011

Declaration by Supervisors

This thesis has been submitted for examination with our approval as University of Eldoret Supervisors

…………………………… Date …………………………

Prof. K.M. Khanna
Department of Physics
University of Eldoret, Kenya

…………………………… Date …………………………

Dr. K.M Muguro
Department of Physics
University of Eldoret, Kenya

DEDICATION

To my family who always motivates me to study.

ABSTRACT

The correlation between nucleon-nucleon interactions $V_{(r)}$ and the pairing energy gap Δ is studied theoretically. When two nuclear particles collide due to the interaction, they get scattered after approaching within a distance of the nearest approach or scattering length a which leads to deviation from original path and this leads to phase shift δ_l. Hence there will be a relation between δ_l, a and $V_{(r)}$. This is what has been calculated by assuming a particular form of interaction potential $V_{(r)}$, and in this case Yukawa potential has been used. The theoretical calculations leads to a singlet $^1S_0$ scattering length of $a = -18.3\ fm$ which is very close to the value of $a = -19.3\ fm$ reported in the literature. Working on the bare nucleon-nucleon interaction in $^1S_0$ channel calculations were done using Yukawa potential, which shows that phase shift $\delta_l(k_F)$ is directly related to Fermi momentum k_F.

TABLE OF CONTENTS

Contents

DECLARATION .... **ii**
DEDICATION .... **iii**
ABSTRACT .... **iv**
TABLE OF CONTENTS .... **v**
LIST OF SYMBOLS .... **vii**
ACRONYMS .... **viii**
LIST OF TABLES .... **ix**
LIST OF FIGURES .... **x**
ACKNOWLEDGEMENT .... **xi**
INTRODUCTION 1
1.1. Introduction 1
1.2. Atomic nucleus 1
1.3. Nature of nuclear forces 2
1.4. Statement of the problem 2
1.5. Objectives of the study 3
1.5.1 General Objective 3
1.5.2 Specific Objectives 3
1.6. Significance of the study 3
CHAPTER TWO **5**
LITERATURE REVIEW 5
2.1. Introduction 5
2.2. Nucleon-Nucleon Interaction 7
2.3. Super fluidity in neutron star matter 8
2.4. Binding energy 9
2.5. Binding energy of the nucleus 10
2.5.1. Semi-Empirical Mass Formula based on Liquid drop model. 10
2.5.2. Volume Energy 11
2.5.3. Asymmetry Energy 11
2.5.4. Surface Energy 12
2.5.5. Coulomb Energy 12
2.6. Pairing Energy 12
METHODOLOGY **14**

3.1. Introduction 14

3.2. The Relation between nucleon-nucleon interactions and the pairing energy gap 17

3.3. Nucleon-nucleon interaction Potentials 23

3.4. Relation to phase shifts using Yukawa interaction potentials 23

CHAPTER FOUR 31

RESULTS AND DISCUSSION 31

4.1. Introduction 31

4.2. Results of pairing energy gap and Fermi momentum for $a_{nn} < 0$ 31

4.3. Results of pairing energy gap and Fermi momentum for $a_{nn} > 0$ 32

4.5. Relation between the Phase shift and Fermi momentum using 34

CHAPTER FIVE 36

CONCLUSIONS AND RECOMMENDATIONS 36

5.1. Conclusions 36

5.2. Recommendations 38

REFERENCES 39

APPENDICES 42

Appendix II : Similarity Report 47

LIST OF SYMBOLS

a_o	Nuclei radius
a, a^+	Annihilation and creation operator
a_{nn}	Scattering length
Δ	Pairing Gap
A	Mass number
B	Binding energy
C	Specific heat capacity
c	Speed of light
E_n	Total energy
f	Binding fraction
G	Pairing interaction
$\bar{G}$	Neutron-proton interaction
H	Hamiltonian
H_o	Unperturbed Hamiltonian
H'	Perturbed Hamiltonian
h	Planck's constant
k_B	Boltzmann's constant
M	Mass of the nucleus
m_n	Neutron mass
m_p	Proton mass
N	Neutron number
P	Momentum operator
R	Nuclear radius
T	Kinetic energy operator
T_C	Transition temperature
V	Potential energy
Z	Atomic mass
β, γ	Perturbation parameters
ε_F	Fermi energy
η	Neutron excess parameter
v	Many-body potential energy operator

ϕ Wave-function

ε_i Single-particle energy

ACRONYMS

BCS Bardeen Cooper-Schrieffer

CSB Charge Symmetry Breaking

LEDF Local-Energy-Density-Functional

QCD Quantum Chromo-Dynamics

SEMF Semi-Empirical Mass Formula

LIST OF TABLES

Page

1. Table A1: Coupling strength and density gradient term 33

2. TABLE A2: Fermi momentum scattering length and pairing gap 34

3. TABLE A3: Scattering length and pairing gap 35

4. TABLE A4: Phase shift and Fermi momentum 36

5. TABLE A5: Fermi momentum and energy gap 37

6. TABLE A6: Physical constants 38

LIST OF FIGURES

Figure 4.1: Variation of pairing energy gap with scattering length for $a_{nn} < 0$

Figure 4.2: Variation of pairing energy gap with scattering length for For $a_{nn} > 0$

Figure 4.3: Variation of pairing gap with Fermi momentum using Yukawa potential

Figure 4.4: Variation of Phase shift with Fermi momentum using Yukawa potential

ACKNOWLEDGEMENT

I sincerely thank my supervisor Professor K. M. Khanna for his creativity, unique and helpful discussion which inspired me to work on this thesis. He not only inspired me to work on my research but also introduced me to nuclear energy reactor which will add some Megawatts to our national grid system. I also thank Dr. Muguro for his contribution and much more directions on my thesis writing and research. Not forgetting School of Science and Department of Physics for giving me opportunity to further my studies in Master of Science in Physics.

CHAPTER ONE

INTRODUCTION

1.1. Introduction

Two nuclear particles within the range of nuclear force scatter when they are within the scattering length a or the impact parameter. Depending upon the direction of scattering, there will be a phase shift δ_l and this will certainly depend on the nature of interaction potential $V_{(r)}$.

1.2. Atomic nucleus

At the beginning of nineteenth century, it was proposed that the atoms of different elements were built up of different numbers of hydrogen atoms, but the elements like chlorine and copper do not have whole number masses. This hypothesis was then abandoned. It was then proposed that the charge of the nucleus is the same as the charge on the electrons revolving in Bohr orbits. For Z electrons, the charge of the nucleus will be Ze-positive charge, so that the atom as a whole is neutral. This concept was based on the Rutherford model of the atom when he concluded, by conducting $\propto$- scattering that the positive charge of the nucleus is concentrated at the center of the nucleus. But the concept of a nucleus being composed of protons could not explain the large increase in the masses of the nuclei (or atoms) as the mass number A increased. The number of electrons and protons were equal (Z), but the mass number A was very different from the proton number Z. Then came the discovery of a neutral particle by (J. Chadwick 1932), and this particle was called neutron. Neutron was considered as a new type of elementary particle along with proton and electron. Its mass (m_n) is slightly more than the mass of the proton$\left(m_p\right)$. All these three particles are fermions and have spin $\frac{1}{2}$.

1.3. Nature of nuclear forces

A nucleus is basically composed of neutrons and protons that are very strongly bound to each other by so called short-range saturated nuclear forces. By the word saturated it means that there is no force which is much bigger than the gravitational and electromagnetic forces. None of the forces can account for the large nuclear binding per nucleon (Preston 1962, Bardeen & Schrieffer 1957). The range of the nuclear force is of the order of 2 fm, and within this range the nuclear force is very strongly attractive. The smallest nucleus compose of a neutron and a proton called deutron, its binding energy is small since it is difficult to distinguish between the core region and surface region of the nucleus, the proton and neutron may be near the surface such that they are not that strong bound. This is true of small a nuclei, roughly up to $\cong 14$.

1.4. Statement of the problem

When two nuclear particles come together such that they will be in the range of the force, they will interact as they scatter. The direction of scattering depends on the nature and strength of the interaction potential V(r), and there will be a phase shift δ due to the process of scattering. Another parameter involved in scattering is the distance of nearest approach, also called scattering length a, and the sign of a depend on the sign of V(r), i.e. whether the potential or force is attractive or repulsive. In finite nuclei, infinite nuclear matter (whether symmetric or asymmetric), and or neutron matter (as in neutron stars), the systems could be in a superfluid state when the particles interact in pairs and the state of the pair is $^1S_0$ (singlet state). This leads the existence of a pairing energy gap Δ in the excitation spectrum of the system. Now since the pairing energy gap Δ is a consequence of the existence of the inter-particle interaction V(r) and the parameter a and δ are also related to V(r); hence there must exist some relationship between V(r), a, δ and Δ, but has not be investigated in detail using different parameters.

1.5. Objectives of the study

1.5.1 General Objective

The general objective of this study was to investigate the correlation between nucleon-nucleon interaction, pairing energy gap and phase shift for identical nucleons in a nuclear system.

1.5.2 Specific Objectives

The specific objectives of the study were:

1) To establish a relation between scattering length a_{nn} and pairing gap Δ.
2) To obtain numerical values of pairing gap Δ using lead isotopes and compare with experimental and theoretical values.
3) To establish correlation between Energy gap against and Fermi momentum using Yukawa potential.
4) To determine the relation between energy gap Δ, nucleon-nucleon interaction potential V_r and phase shift $\delta_{l(k)}$, using Yukawa potential.

1.6. Significance of the study

Assuming pair interaction in finite nuclei and nuclear matter, the objective is to correlate various parameters, and establish how there is an agreement between theoretical calculations and experimental observations. It is shown theoretically how nucleon-nucleon interactions influence the pairing gap, scattering length and phase shift. The correlation between different parameters, like energy gap Δ phase shift δ_l, scattering length a_{nn} and the strength of the interaction potential has been established. Since interactions between the nucleons play a predominant role in determining the binding energy of nucleons and nuclei, it is important to understand the role played by the inter-nucleon force or potential in determining nuclear properties. Experimental study of the scattering of nuclear particles involves the study and measurement of scattering length a and the phase shift δ_l. Hence it is appropriate to study how these parameters depend on each other, and this is the main purpose of the theoretical study presented in this thesis. Calculations on pairing energy gap are of great importance in describing the pairing of nucleons. The scattering of nucleons is influence by interaction potential which contributes to the stability of the nuclei.

CHAPTER TWO

LITERATURE REVIEW

2.1. Introduction

Neutron-neutron pairing, neutron-proton pairing and proton-proton pairing is the basic to the study of the properties of finite nuclei and nuclear matter. Quantum many-body theory is used to study the properties of finite nuclei and nuclear matter, especially symmetric nuclear matter. The supper-fluidity of nuclear matter, finite nuclei and neutron stars has been studied from time to time (Khanna, 1962 and Heisenberg. & Jensen, 2000). The main focus in this study is the link between supper-fluidity of nuclear matter symmetric and asymmetric and the role played by Yukawa in nucleon-nucleon interactions. In the low density systems (outer part of the star) supper-fluidity (in the case of a star it is neutron supper-fluidity) is expected mainly in the attractive singlet $^1S_0$ channel. Low density corresponds to values of the order of $\rho \sim \frac{\rho_0}{10}$ where ρ_0 the highest is or saturation density of symmetrical nuclear matter.

At low density, average inter particle distance is large and thus the particles (neutrons or protons) mainly experience the attractive components of the $^1S_0$ pairing interaction. But, at higher densities of the order of ρ_0, the pairing effect is compensated by strong short-range repulsive components of the interactions. Supper-fluidity in a system is exhibited by the appearance of an energy gap Δ, in the energy spectrum of system. But how the density ranges affects the magnitude of the gaps remains uncertain (Dean & Jensen. 2003), which means particularly when nucleons are in a super-fluid level of state.

In infinite nuclei and nuclear matter, coulomb forces are weak, and then due to the charge symmetry the nuclear forces between neutrons and neutrons, protons and proton, neutrons and protons are assumed to be identical. This also results in isotopic differences of the nucleon-nucleon interactions, with charge symmetry; the word nucleon is introduced irrespective of the fact whether the nuclear particle is proton or neutron. Such a system of nucleons is described by the quantum number called the iso-spin or isotopic spin quantum number, and since the nucleons are fermions, the iso-spin $\tau = \frac{1}{2}$ (half-integral spin). This spin can have two fundamental values, $-\frac{1}{2}$ and+ $\frac{1}{2}$ and these will describe the two states of the nucleons. The proton is described by $\tau = -\frac{1}{2}$, and the neutron is described by $\tau = +\frac{1}{2}$. The iso-spin quantum number is devoted in general, by the symbol τ. If N is the number of neutrons and Z is the number of protons in a nucleus, then the projection T_z of the iso-spin is written as

$$T_z = \frac{N-Z}{z} \tag{2.1}$$

In a nucleus with equal number of neutrons and protons $T_z = 0$. For a nucleon-nucleon system with $T = 1$, the iso-spin projections are $T_z = +1,0,-1$. here $T_z = +\frac{1}{2}+\frac{1}{2}$ will correspond to neutron-neutron system. The $T_z = 0 = +\frac{1}{2}-\frac{1}{2}$ will correspond to a neutron-proton systems; and $T_z = -1 = -\frac{1}{2}-\frac{1}{2}$ will correspond to a proton-proton system. Hence the total spin angular momentum $J = +1 + 0 - 1 = 0$ and this maintains the anti-symmetry of the total nucleon-nucleon wave-function. Similarly when $T = 0 = \frac{1}{2}-\frac{1}{2}$, it will refer to a neutron-proton system. This arrangement corresponds to two different types of elementary particles pairing and total spin will depending on spin and isotopic spin quantum numbers of proton-neutron etc. pairing involved. More details can be found in (Ghosh 2010 and Blatt & Weiss Kopf. 1982) theoretical nuclear physics.

2.2. Nucleon-Nucleon Interaction

The nucleon-nucleon interaction or the nuclear force is a very strong short-range force whose range is the order of 1.5 -2.0 fm. It is stronger that it is very large when compared to the gravitational and electromagnetic force. When two nucleons or any nuclear particles approach each other two specific physical parameters will be involved. One is the distance of nearest approach a, and the other is the range of the nuclear force r_o. The particles will interact and scatter only if the distance of nearest approach which is also called impact parameter $a < r_0$. If $a > r_0$ then the particles will not feel the interaction potential. Hence there has to be some correlation between the interaction potential and the impact parameter (also known as scattering length). There are many types of the nucleon-nucleon interaction potentials, and hence they will lead to varying magnitudes for a and r_o.

When the two nucleons interact and scatter, the outgoing waves are affected in phase or in amplitude or in both. Hence, there will be existence of some definite correlation between the interaction of the potential V(r) and phase shift δ_l . In the scattering or interaction process, the scattering length a must also be involved. Therefore the interaction potential V(r), the scattering length a and the phase shift δ_l must be correlated to each other.

In general low-energy scattering experiment can measure only two parameters of the potential, namely the scattering length ‘a’ and the range r_o. And the relation between a, r_o and the phase shift δ_0(δ_l is the phase shift for the angular momentum state l, and δ_o will be the phase shift for the s-wave phase shift when $l = 0$ and $s = 0$) is (Blatt & Weiss. 1982)

$$k \cot \delta_0 = -\frac{1}{a} + \frac{1}{2} r_0 k^2 \quad (2.2)$$

Where k is the momentum transfer in the scattering process.

In superfluid systems, the wave function that describes pairing in the superfluid state is the well-known BCS (Bardeen Cooper Schrieffer 1957) wave function written as

$$I\emptyset_0 >^{BCS} = {}_{j,m>0}^{\pi}(U_j + V_j . a_{j,m}^{+} a_{-j,-m}^{+}) 1.0 >$$

(2.3)

Where U_j=Probability amplitude of the state j not occupied by a pair of particles

V_j= probability amplitude of the state j occupied but a pair of particles with equal opposite momenta and spins10>= vacuum state of real particles such that $a_{\bar{j}m} 10 \geq 0$, where $a_{\bar{j}m}$ is a destruction operator $U_j and\ V_j$ Are real constants of transformation such that $U_j^2 + V_j^2 = 1$ for fermions.

Using this wave function and the pairing Hamiltonian for a nuclear system, an expression can be derived that correlates pairing energy gap Δ with the interaction potential V(r) and the Fermi momentum k_F (Fetter. and Walecka. 2003). It is found that the pairing energy gap at the Fermi surface depends on the coherence length ξ of the cooper pairs and the value of ξ depends on the density of matter that in turn depends on the state of confinement of the superfluid system.

2.3. Super fluidity in neutron star matter

In both the $^1S_0$ and$^3P_2$-$^3F_2$ channels phase shifts shows how the nucleon-nucleon interaction is in the attractive side of the force. $^1S_0$ channel, neutron-neutron channel firmly recommends there must be a pairing condensate for low density. In $^3P_2$-$^3F_2$ channel the attraction potential is strongly attractive at higher energy densities. Different groups have obtained results which are in close agreement with $^1S_0$ pairing energy gap values. The highest value of about 3 MeV is obtained at Fermi momentum of the order of $K_F \approx 0.8 fm^{-1}$ (Khanna 1962). In the state $^3P_2$-$^3F_2$ pairing gap is important for cooling of neutron stars, where the gap affects the cooling with a factor $e^{-\Delta/T}$, where Δ the pairing energy gap, and this is severe for temperatures below the pairing energy gap. The results obtained long ago along with modern potentials are in agreement at low densities below $k_F \cong 2.0 fm^{-1}$ but differ at higher densities. This shows that, within $E_{Lab} \approx 350 MeV$ the scattering data makes the ambiguities in the results for the pairing energy gap. To workout the pairing energy gap for high densities above $k_F \cong 2.0 fm^{-1}$ various potentials at high energies outside the range are required to be fitted into the scattering data. The results might depend on the model chosen resulting in the differences among the predictions at higher densities.

2.4. Binding energy

Available nuclear force theory should be able to predict binding energy of heavy nuclei more accurately. The binding energy is normally calculated by employing the Hatree-Fock theory and the t-matrix elements calculated with harmonic oscillator basis. Yale and BJ, with which most calculations for finite nuclei have been done, are found to under bind the nuclei by a large amount.

The static interaction (Hassan and Ramadan 1978) is also used to approximately calculate the binding energy. The parameters that are used in the interaction are fixed by using the data on the light nuclei. Generally modifications are made to the nuclear force to make it yield more correct binding energy according to perturbation calculation. In such models, the interaction between the nn pairs and the excess neutrons in the neutron skin is considered as a perturbation for calculations in heavy nuclei so as to ascertain the effects on the binding energy.

2.5. Binding energy of the nucleus

Binding energy per nucleon (Khanna 1968), increases for light nuclei; for A less than 10 the values for f are smaller and irregular. As A increases f stays at about 8.5 MeV, from $A = 50$ and $A = 150$. For heavy nuclei the value of binding fraction f falls to about 7.5 MeV for Uranium 238. The small value of f for light nuclei is due to the surface effect, while the small value of f for heavy nuclei is due to proton repulsive forces. Heavy nuclei have a large number of protons $Z,$ thus there is a sufficient coulomb repulsion. This weak force is strong enough to cause a reduction in binding fraction which is proportional to $Z^{1/3}$.

The binding energy B of the nucleus consisting of Z protons and $N = A - Z$ neutrons is defined implicitly in the atomic mass unit

$$M(A,Z)c^2 = [Zm_p + (A-Z)m_n]c^2 - E_{nuclear} = \Delta mc^2$$

(2.4)

Where Δ is the mass excess or mass defect, M is measured in atomic mass unit (amu). Greater the stability of a nucleus implies greater binding energy per nucleon. The binding energy for heavy mass nuclei grows as A increases. The variation with respect to A suggests that the force is saturated and the effect of the interaction which is only felt in neighborhood of the nucleon.

2.5.1. Semi-Empirical Mass Formula based on Liquid drop model.

The mass of the nucleus is define as

$$M(A,Z) = Zm_p + (A-Z)m_n - \frac{B(A,Z)}{c^2}$$

(2.5)

Calculation of nuclear binding energy begins with the nuclear many-body Hamiltonian. The actual energy in the Bethe-Weizsacker mass formula is given as;

$$M(Z,N)c^2 = \left(M_p Z + M_n N\right)c^2 + E(Z,N)$$

(2.6)

Where, $M(Z,N)$ is the mass of the nucleus made up of Z protons and N neutrons and $E(Z,N)$is the binding energy of the nucleus. For a stable nucleus E is negative and hence binding energy is positive, the quantity of E consists of five different components

2.5.2. Volume Energy

Considering a nucleus made up of an equal number of protons and neutrons with no coulomb interaction among its protons forms the basis of volume energy term. The volume energy is a consequence of the nuclear many-body Hamiltonian containing only the kinetic energy and nucleon-nucleon interactions.

$$E_V = -a_v A \tag{2.7}$$

This provides attractive energy contribution to the total binding energy

2.5.3. Asymmetry Energy

The asymmetry energy is a consequence of the difference between proton and neutron number. In the absence of Coulomb interaction between protons, a nucleus may be composed of equal numbers of protons and neutrons. To create the observed neutron excess, nucleons are to be shifted from proton side to neutron side of the Fermi gases. These neutrons can be added above Fermi level only and this means, energy has to be added into the system. Nuclear binding is reduced by asymmetric energy. In symmetrical nuclear system similarly the same energy is required to shift the nucleons if proton excess is required.

$$E_A = \frac{1}{4} a_v x_v \frac{(N-Z)^2}{A} \tag{2.8}$$

2.5.4. Surface Energy

The correction to the negative volume energy due to the surface effect is a term with a positive sign and must be proportional to the surface area of the nucleus. The surface energy depends on A explicitly and proportional to $A^{2/3}$, thus

$$E_s = +a_s A^{2/3} \tag{2.9}$$

2.5.5. Coulomb Energy

The Coulomb energy term is due to the Coulomb repulsion between $\frac{Z(Z-1)}{2}$ pairs of protons in the nucleus. For a spherical nucleus of radius R with the charge spread uniformly throughout the volume of the sphere.

$$E_C = -\frac{3}{5} \frac{1}{[4\pi c_o]} \frac{Z(Z-1)c^2}{r_o A^{1/3}} \tag{2.10}$$

2.6. Pairing Energy

Nucleons (protons and neutrons) have a general tendency to form pairs (pp, nn and np pairs) to create a stable nuclear system under the influence of short-range nucleon-nucleon attractive interaction. The study of nucleon separation energy brings out the role of the pairing energy in determining the binding energy of the nuclear systems (Brown 2013), since the separation energy is defined as the energy required removing a nucleon from a nucleus in the ground state leaving the residual nucleus also in the ground sate. The separation energy S_p for the protons is written in terms of the binding energy of the involved nuclei, i.e.

$$S_p = B(N, Z+1) B(N,\ Z-1,).$$

(2.11)

And similarly the separation energy of the neutron s_n is,

$$S_n = B(N+1, Z) + B(N-1, Z)$$

(2.12)

It is found that these separation energies are different for different nuclei, and their magnitude is also different from the binding fraction f which is binding energy per nucleon. This is due to the fact that in general $N \neq Z(N > Z)$ and that the number of the protons and neutrons could be odd and even in different nuclei. This point to the role played by odd-even, odd-odd, even-even and even-odd effects on the binding energy of the nucleons. Since interactions (Changizi 2015), between the nucleons play a predominant role in determining the binding energy of nucleons and nuclei, it is important to understand the role played by the inter-nucleon force or potential in determining nuclear properties.

However, to study the properties of finite nuclei and nuclear matter, other nuclear models, namely, liquid drop model, Fermi Gas Model, Shell Model etc. were proposed. But minimum information about the characteristics of nuclear particles could be extracted from the semi-Empirical mass formula (liquid Drop Model) and Shell Model only. Experimental study of the scattering of nuclear particles involves the study and measurement of scattering length a and the phase shift δ_l. Hence it is appropriate to study how these parameters depend on each other, and this is the main purpose of the theoretical study presented in this thesis.

CHAPTER THREE

METHODOLOGY

3.1. Introduction

When the particles approach each other particle, and they will be within a field of force to one another which means interaction and scattering will occur in process three factors and parameters play an important and significant role; the distance of nearest approach, called scattering length a, Phase shift δ due to angular scattering and the interaction potential $V(r)$ between the two particles

Working using large finite nuclei symmetric and asymmetric infinite nuclear matter and neutron matter (in stars), another important tool get involved and this is called $^1S_0$ pairing energy gap Δ. Hence there exist, a definite relationship of interaction potential $V_{(r)}$, scattering length a , Phase shift δ and the pairing energy gap Δ.

This is exactly what has been studied in this thesis by choosing Yukawa interaction potential, the relation of phase shift and Yukawa interaction potential, the relation between pairing energy gap Δ and scattering length a and the relation between phase shifts δ with Fermi momentum k_F has been done. The magnitude of the pairing energy gap will be dependent on the strength of the nucleon-nucleon interaction. The s-wave scattering and its length and the value of δ and k_F. The relevant relations between the pairing energy gap Δ, the phase shift δ_l, the interaction potential $V_{(r)}$ and Fermi momentum k_F are derived and used in the calculations.

The nuclear pairing forces strongly determine the low energy nuclear-structure properties. It is important to consider pairing effects in the calculation of nuclear masses, β strength functions, low lying quasi-particle spectrum that contains pairing energy gap and other quantities that depend on the low-energy microscopic structure of the nucleus. For obtaining realistic results, an appropriate choice of pairing model with realistic inter-particle interaction potential V(r) and other likely parameters involved has to be chosen. The other parameters are the distance of nearest approach or scattering of length a, the possible phase shift δ on scattering, and pairing energy gap in quasi-particle energy spectrum along with the Fermi momentum k_F all that has been mentioned above, when put together constitutes, the pairing model and the pairing parameters that can be used to study as to how one parameter varies with the other parameter.

When two nucleons come together to form a bound pair they consume what is known as pairing energy. Now if a nucleon is to shift to an energy state in the excitation spectrum, or it has to occupy an excited energy state, the pair has to be broken, and to do this the minimum energy required is the pairing energy. Thus in the energy spectrum there is no state between the lowest state in which the pair lies before breaking, and the excited state to which nucleon shift after breaking. This minimum energy between the lowest state and excited state (Chadian and Sabatier, 1992) is known as the pairing energy or pairing energy gap.

Since neutrons and protons are fermions, they are distributed among the energy levels up to the top of Fermi surface. The position of Fermi surface or Fermi levels depends on the number of neutrons and protons. Since in heavy nuclei number of neutrons $N > Z$ number of protons, the Fermi energy E_{FN} for neutrons is more than the Fermi energy E_{FZ} for protons. For even-even nuclei, the nucleons will be in pairs and the pairing energy or pairing energy gap will have a finite value. Since the pairs near the Fermi surface can take part in the process of excitation, the energy gap will depend on the Fermi momentum k_F. Thus if Δ is the energy gap, then we can write it as $\Delta(k_F)$. Whereas in the case of odd mass number nuclei, there will be an unpaired nucleon near the Fermi surface, the excitation energy spectrum will not have any energy gap since no breaking of pair is involved, the unpaired nucleon can easily get excited.

In the case of symmetric nuclear matter, when the number of neutrons N= number of protons Z, and the number of nucleons in the system is very large ($N \to \infty, V \to \infty$, such that the density $\rho = \frac{N}{V}$ =constant), the energy in excitation spectrum will have a finite energy gap $\Delta(k_F)$.

When two nucleons approaches each other, they scatter resulting in phase shift δ, and the phenomena of scattering depends in the distance of the nearest approach called scattering length a, and the magnitude and the sign of the interaction potential $V(r)$. Hence δ, a and $V(r)$ must have some definite correlation between them. It is this correlation that has been studied in this thesis using Yukawa potential (Yukawa 1985) as $V(r)$

In the system of degenerate fermions, the Fermi surface is unstable due to the formation of pairs (between the nucleons), and the pairs are formed due to the attractive interaction between the interacting particles with spin and momenta in the opposite direction($a^+_{\uparrow k} a^+_{\downarrow(-k)}$). As a result of the instability, according to the BCS theory, there is a collective reorganization of particles at energies near and around the Fermi surface or Fermi energy, and this leads to the appearance of an energy gap in quasi-particle spectrum of nucleons. Such reorganization manifest itself in the formation of what is known as cooper pairs (Bardeen, Cooper & Schrieffer 1957).

3.2. The Relation between nucleon-nucleon interactions and the pairing energy gap

Different types of nuclear systems have varying compositions. Light stable nuclei have practically equal numbers of neutrons (N) and protons (Z) except when we deal with isotopes. As the nuclear mass number A increases neutron number (N) increases faster than the proton number (Z) and in general in heavy nuclei N>Z. Then there is symmetric nuclear matter in which the number of neutrons is assumed to be equal to the number of protons; and the number of nuclear particles, neutrons and protons is assumed to be infinitely large ($A = Z + N \rightarrow \infty$). Properties of asymmetric nuclear matter in which $N \neq Z$, and in general N>Z has also been studied. Recent astrophysical studies have confirmed the existence of neutron stars which are composed of 90% or more neutron matter.

It should now be clearly understood that the pairing interaction, rather pairing interaction matrix element G (interaction between a pair of nuclear particles) between the two nuclear particles cannot be the same for different types of nuclear systems. In finite nuclei the neutron-neutron (N-N) pairing interaction G, and proton-proton (p-p) and neutron-proton pairing interactions will not be identical. The value of G will be different in different nuclei since in finite nuclei $N \neq Z$, and in general N>Z.

The energy excitation spectrum differs from nucleus to another nucleus and from one nuclear system to other nuclear systems. The spectrum depends on the magnitude of the pairing interaction G and since G is not the same for all nuclei. The excitation spectrum is different for different nuclei. Since there exists energy gap Δ (also called pairing energy gap) in the spectrum the energy gap Δ and the pairing interaction G must have some correlation, i.e. Δ, depends on G. For nuclei with N >Z, The isotopic spin τ plays an important role in determining the value of G, and since Δ depend on G, it can be summarized that Δ will depend on the isotopic spin τ. It is found that Δ reduces with finite isotopic spin τ.

Similar concepts will be applicable when we deal with symmetric nuclear matter; asymmetric nuclear matter or neutron matter ultimately Δ has to depend on pair on pair interaction or interaction potential between a pair of particles. Hence it would be appropriate to study the dependence of Δ on G or V(r), the interaction potential between a pair of particles separated by a distance r.

Pairing energy gap equation is obtained by using two-body Hamiltonian $\hat{H}$

$$\hat{H} = \hat{H}_1 + \hat{H}_2$$

(3.1)

Where

$$\hat{H}_1 = \sum_\alpha \epsilon_\alpha \; a_\alpha^+ a_\alpha$$

(3.2a)

$$\hat{H}_2 = \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} \; a_\alpha^+ \, a_\beta^+ a_\delta a_\gamma$$

(3.2b)

Here a is fermion annihilation operator and a^+ fermion creation operator V is uncoupled matrix elements of the two- body interaction. $\hat{H}_1$; is the kinetic energy of the assembly of fermions, the sums stands for possible single- particle quantum numbers.

The Hamiltonian contains a pairing type inter-particle interaction as in the case of Fermi-gas model with two fold degeneracy spin of a particle is $S = \frac{1}{2}$, the degeneracy of the single- particle levels is $2S+1=2$. When two particles have spin up $(\uparrow)$ and down $(\downarrow)$, the net spin $S=\frac{1}{2}-\frac{1}{2}=0$ and continued with $l=0$ relative angular momentum state (Chadian and Sabatier, 1992), $J=0=l+S$, and it is called singlet $^1S_0$ state; and interacting pair of particles will be in this state. For $l\neq 0$, the partial wave states are due to Yukawa interaction potential V(r). The interaction is determined via scattering length a at low densities. This allows excited spectrum to be expressed as expansion in terms of energy density. For the neutron-neutron scattering in the $^1S_0$ channel, the scattering length a in $^1S_0$ channel is $a=-18.8\ \pm 0.3fm$ (Dean & Jensen 2003).Thus for single-particle energies, the scattering length can be approximated (which means particle interact in pairs known as pairing approximation) that will lead to the following approximation for two body Hamiltonian Eq. (3.1) Thus we can write

$$H=\sum_i \epsilon_i\ a_i^{+}a_i+\frac{1}{2}\ G\sum_{ij>0}\ a_{i\uparrow}^{+}\ a_{i\downarrow}^{+}a_{j\downarrow}a_{j\uparrow}$$

(3.3)

Where indices *i* and *j* goes over the number of energy levels *L*. The arrows $\uparrow and\ \downarrow$ refer to the time- reversed state, strength of the pairing force is G, 2 in the denominator is due to the fact that in pairing each particle is counted twice, so we must divide the term by 2, and ϵ_i represents the single particle energy of the levels *i*. Using the properties of infinite nuclear matter with identical particles,

$$\sum_k \rightarrow \frac{V}{(2\pi)^3}\int_0^\infty d^3\,k$$

(3.4)

Rewriting Eq. (3.3) gives,

$$H=V\sum_{\sigma=\pm}\int\frac{d^3k}{(2\pi)^3}\epsilon_{k\sigma}\ a_{k\sigma}^{+}a_{k\sigma}+\ GV^2\int\frac{d^3k}{(2\pi)^3}\int\frac{d^3k'}{(2\pi)^3}\ a_{k+}^{+}\ a_{-k-}^{+}a_{-k'-}a_{k+}$$

(3.5)

The first terms in Eq. (3.5) represents the kinetic energy of the assembly of particles, with $\epsilon_{k\sigma} = \frac{k^2}{2m}$, the label $\sigma = \pm\frac{1}{2}$ stands for the spin of the particles, and *V* is the volume of the container containing the particles. The predicted value of the two body interaction having constant interaction strength *G* that pair of particles is given in the second term energy.

The energy gap $\Delta_{(k)}$ in non-finite nuclear matter will obtained by using the BCS equation for the energy gap equations, for a model (Blatt & Weiss 1982) and (Fetter and Walecka 2003) in which (neutron-neutron of scattering in $^1S_0$ channel -$^1S_0$ pairing), finally it gives,

$$1 = \frac{GV}{2(2\pi)^3}\int_0^\infty dk'\, {k'}^2\, \frac{1}{E(k')} \qquad (3.6)$$

Where, $E_{(k)}$ is the quasi particle energy given by

$$E_{(k)} = \left\{\left[\epsilon_{(k)} - \epsilon_{(k_F)}\right]^2 + \left[\Delta_{(k)}\right]^2\right\}^{\frac{1}{2}}$$

(3.7)

Where $\epsilon_{(k)}$ is the single-particle energy of a neutron with momentum *k*, and $k_F$is the Fermi momentum, free single-particle energies such as $\epsilon_k = \frac{k^2}{2m_N}$($m_N$= mass of the neutron) is used.

However, if there exists an effective single-particle potential δ, then

$$\epsilon_k = \frac{k^2}{2m_N} + \delta$$

(3.8)

Then an analyzed expression for pairing energy gap for low density limit (Etrin 2017) is obtained after combination of Eq. (3.7) and the equation of scattering length *a* and its correlation to interaction, to give equation,

$$-\frac{m_N GV}{4\pi a} + 1 = -\frac{GV}{2(2\pi)^3}\int d^3k' \frac{1}{\sqrt{\left[\epsilon_{(k)} - \epsilon_{(k_F)}\right]^2}},$$

(3.9)

The integral on the right hand side of Eq. (3.9) is divergent at $\epsilon_{(k)} = \epsilon_{(k_F)},$ However, by subtracting Eq. (3.6) from Eq. (3.9) gives,

$$\frac{M_N}{4\pi a_0} = -\frac{1}{2(2\pi)^3}\int d^3k \left[\frac{1}{E_{(k)}} - \frac{1}{\sqrt{\left[\epsilon_{(k)} - \epsilon_{(k_F)}\right]^2}}\right],$$

(3.10)

Eq. (3.9) is now not divergent and interaction strength G cancels on both the sides. Using dimensional regularization techniques, the following analytic expression was obtained.

$$\frac{1}{k_{(F)}a_{nn}}(1+x^2)^{\frac{1}{4}}P_{\frac{1}{2}}\left(-\frac{1}{\sqrt{1+x^2}}\right) \qquad \text{Where} \quad x = \frac{\Delta_{(k_F)}}{\epsilon_{(k_F)}}$$

(3.11)

And $P_{\frac{1}{2}}$(x) is the Legendre function for the given Fermi momentum K_F thus will give the pairing energy Δ equation, for smaller values of $k_F a_{nn}$ in the form (Dean & Jensen 2003).

$$\Delta = c\epsilon_F \exp\left(\frac{\pi}{2k_F a_{nn}}\right),$$

(3.12)

This is a consequence of comes about by the behavior of $P_{\frac{1}{2}}(x)$ which will have logarithmic singularities (Schiff.1968) at $z = -1$. For larger values in $k_F a_{nn}$, the gap is proportional to energy at Fermi surface $\epsilon_{(k_F)}$, approaching $\Delta \cong 1.6\epsilon_{(k_F)}$. Eq. (3.12) allows us to get the value of the gap at Fermi surface $\Delta(k_F)$, knowing experimental parameter, the scattering length a_{nn}, at low densities with λ as a constant, $\Delta(k_F)$ can be calculated.

Singlet scattering length Equation (Dean & Jensen 2003), is given by

$$a_{nn} = \frac{\pi}{2k_{0F}}\left(\frac{\sqrt{2m\epsilon_c}}{\hbar k_{0F}} + \frac{2}{f_{ex}^{\varepsilon}}\right) \equiv \frac{\pi}{4k_{0F}}\left(\frac{1}{f_{ex}^{\varepsilon}} - \frac{1}{f_{cr}^{\varepsilon}}\right)^{-1}$$

(3.14)

Where critical constant $f_{cr}^{\varepsilon} = -\frac{2k_{0F}}{k_{oc}} \cong -1.912$ and f_{ex}^{ε} is its value in the vacuum (Cottingham Lacombe & Loiseau 1973).

The Pairing energy Gap equations (Chadian, K and P. C. Sabatier, 1992) used

i) low density

$$\Delta = c\epsilon_F \exp\left(\frac{\pi}{2k_F a_{nn}}\right) \qquad For\ a_{nn} < 0$$

(3.15)

Where $c = 8e^{-2}$, $k = k_F$ and $\epsilon_F = \frac{\hbar^2 k_F^2(\rho)}{2m}$

ii) At low density (Elpelbaum, Hammer & Meiner 2009)

$$\Delta = \frac{\hbar^2}{m}\left(\frac{2\pi\rho}{a_{nn}}\right)^{\frac{1}{2}}, \qquad For\ a_{nn} > 0$$

(3.16)

Here: m is the mass of free nucleon.

3.3. Nucleon-nucleon interaction Potentials

There are various potentials that can be used to obtain the values of energy gap due to scattering. A number of nucleon potentials have been proposed from time to time among these are;

1. Yukawa potential (Yukawa 1985)

$$V_{(r)} = \frac{\beta}{r} \mathrm{e}^{-\frac{\mathrm{r}}{\beta}} \tag{3.17}$$

2. Hamada-Johnson Potential (Hamada and Johnston 1992)

$$V = V_c(r) + V_t(r)S_{12} + V_{ls}(r)\vec{L}.\vec{S} + V_{ll}(r)L_{12} \tag{3.18}$$

3. Yale-Group Potential (Gross, Cohen, Epelbaum & Machleidt 2011)

$$V = V_{OPEP}^{(2)} + V_c(r) + V_t(r)S_{12} + V_{ls}(r)\vec{L}.\vec{S} + V_{ll}(r)L.S + V_{ql}(r)[Q_{12} - (\vec{L}.\vec{S})^2 \tag{3.19}$$

4. Ried68 Potential (Reid 1988)

$$V = V_c(r) + V_t(r)S_{12} + V_{ls}(r)\vec{L}.\vec{S} \tag{3.20}$$

5. Paris-Group Potential (Lacombe, Loiseau & Richard 1973)

$$V(\vec{r}, p^2) = V_0(r, p^2)SS_1 + V_1(r, p^2)SS_2 + V_{ls}(r)\vec{L}.\vec{S} + V_t(r)S_{12} + V_q(r)Q_{12} \tag{3.21}$$

$$where\ SS_1 = \left(\frac{1 - \vec{\sigma_1}.\vec{\sigma_2}}{4}\right) \text{And } SS_2 = \left(\frac{3 + \vec{\sigma_1}.\vec{\sigma_2}}{4}\right)$$

3.4. Relation to phase shifts using Yukawa interaction potentials

With the results obtained from Eq. (3.12), and the experimental values of the phase shifts, the values of pairing energy gap $\Delta(k_F)$ in pure neutron matter can be determine for the nucleon-nucleon interaction. An important feature of $^1S_0$ nucleon-nucleon scattering is larger and negative scattering lengths a_{nn}, indicating presence of approximately bound state of zero scattering energy. The *n*-*n* T-matrix has a pole (singularity) near the bound state (Brown 2013). Now at low energy we can write

$V\,(k,k') \;=\; \lambda v(k)v(k'),$ where λ is a constant, (3.22)

The relation between $\Delta(k)$ and $V(k,k')$ is given by the $^1S_0$ wave in the uncoupled channel and thus we can write (Lacombe, Loiseau & Richard 1973).

$$\Delta(k) \;=-\frac{1}{\pi}\int_0^\infty dk'k'^2\,\frac{\lambda v^2(k')}{E(k')}$$ (3.23)

Now combining Eq. (3.17) and Eq. (3.18), the energy gap equation can be written as

$$1 \;=-\frac{1}{\pi}\int_0^\infty dk'k'^2\,\frac{\lambda v^2(k')}{E(k')}$$ (3.24)

Now since the integral in Eq. (3.19) depends weakly on the momentum, the integral on the right-hand side of this equation depends very weakly on the momentum form of Δ (k), we put $\Delta(k) \approx \Delta_F$ in $E(k)$. Then Eq. (3.17) – Eq. (3.19) indicates that pairing energy gap $\Delta_{(k)}$ is determined by the diagonal elements $\lambda v^2(k)$ of the *n*-*n* interaction. It is important to understand that in scattering theory two-particle potential can be determined from the knowledge of the phase shifts at all energies, solving for rank-one separable potentials (Chadian, and Sabatier 1992) gives,

$$\lambda v^2(k) = -\frac{k^2+k_B^2}{k^2}\frac{\sin\delta(k)}{k}\,e^{-\alpha(k)},$$ (3.25)

The bound state energy E_B for an attractive potential is $E = -k_B^2$, but here $k_B = 0$ However the phase shift $\delta(k)$ is a function of momentum k, while $\alpha(k)$ is given by a principal value integral:

$$\alpha(k) = \frac{1}{\pi} P \int_{-\infty}^{+\infty} dk' \frac{\delta(k')}{k'-k'}$$

(3.26)

Here, the phase shifts have both the negative and positive momenta dependence such that,

$$\delta(-k) = -\delta(k) \tag{3.27}$$

The pairing energy gap $\Delta_{(k)}$ depend on $^1S_0$ phase shifts. This validity has two limitations. First is that, the separable is valid only at low energies, near the pole in the T -matrix. Next we can see from Eq. (3.19) that we need knowledge of the phase shifts $\delta(k)$ at all energies. This in general is impossible since most phase shift studies stop at laboratory energy $E_{lab} = 350\ MeV$.

The sign of phase shift δ changes sign from positive to negative for $^1S_0$ pairing at $E_{lab} \approx 248.5\ MeV$. Since the integral in Eq. (3.18) is peaked near k_F for low values of k_F, knowledge of $v(k)$ from any value k to k_F may be good enough for calculating the value $\Delta_{(k)}$, as the integrand in Eq. (3.17) is strongly peaked around k_F, .

The $^1S_0$ phase shift data given by Nijmegen has been used in the calculations. The value of integral in Eq. (3.20) is obtained by knowing the value of $\lambda v^2(k)$ from Eqs. (3.18) and (3.19), the method used for calculation is described in (Brown and Jackson 1976). The pairing energy gap $\Delta_{(k)}$ for different values of k_F is obtained from Eq. (3.18), by using the approximation $\Delta(k) \approx \Delta_{(kF)}$.

Have shown that the analytic expression for $\lambda v^2(k)$ is,

$$\lambda v^2(k) = -\frac{1}{\sqrt{k^2+\frac{r_0^2}{4}(k^2+\alpha^2)^2}}\sqrt{\frac{k^2+\beta_2^2}{k^2-\beta_2^2}}$$

(3.28)

With,$a^2 = -\frac{-2}{ar_0}$, $\beta_0 \approx -0.0498fm^{-1}$ and $\beta_2 \approx 0.777\ fm^{-1}$.

Using this approximation the phase shift are positive for all energies, since in Eq. (3.20) shows that $\lambda v^2(k)$ is attractive for all k.

The phase shift, $\delta_{l(k)}$for scattering is related to the interaction potential $V(r)$ via the Born approximation, such that,

$$\delta_l = -\frac{2\mu k_F}{\hbar^2}\int_0^\infty V_{(r)} j_l^2(k_F r) r^2 dr$$

(3.29)

The form of the Bessel function in form $j_l(k_F r)$ in Eq. (3.24) for $l = 0$ is

$$j_{0(k_F r)}^2 = \sin^2\left(\frac{(k_F r)}{(k_F r)^2}\right)$$, and in the ground state $k_F r < 1$

(3.30)

In the ground state $\ell = 0$ and thus Eq. (3.24) can be written as,

$$\delta_\ell(k_F) = -\frac{2uk_F}{\hbar^2}\int_0^\infty V(r) j_0^2\,(k_F r) r^2 dr$$

(3.31)

Here $j_0(k_F r)$ was the spherical Bessel function in the order of zero for $\ell = 0$ that is,

$$j_0^2(k_F r) = \frac{sin^2(k_F\,r)}{(k_F\,r)^2} \tag{3.32}$$

Choosing the value of k_F as 0.1fm^{-1}, 0.2fm^{-1}, 0.3fm^{-1}… such that $k_F\ r < 1$ and hence $\sin^2(k_F r) \cong (k_F r)^2$therefore Eq. (3.27) reduces to

$$j_0^2(k_F r) = 1 \tag{3.33}$$

Substituting the value of Yukawa potential, $V_{(r)} = V_0 \frac{\beta}{r} \exp^{-\frac{r}{\beta}}$ and that of $j_0^2(k_F r)$ from Eq. (3.17) in Eq. (3.31) we get

$$\delta_0(k_F) = \frac{2\mu k_F V_0}{\hbar^2} \int_0^\infty \frac{\beta}{r} e^{-\frac{r}{\beta}} r^2 dr$$

(3.34)

Integrating Eq. (3.34) by parts we get,

$$\delta_0(k_F) = \frac{2\mu k_F V_0}{\hbar^2} \int_0^\infty r\, e^{-\frac{r}{\beta}} dr$$

$$= \frac{2\mu k_F V_0}{\hbar^2} \left[r\left(-\frac{1}{\beta}\right) e^{-\frac{r}{\beta}} \right]_0^\infty - \int_0^\infty \frac{1}{\beta} e^{-\frac{r}{\beta}} dr$$

$$= \frac{2\mu k_F V_0}{\hbar^2} + \frac{1}{\beta} \int_0^\infty \frac{1}{\beta} e^{-\frac{r}{\beta}} dr$$

$$= \frac{2\mu k_F V_0}{\hbar^2} \left[\frac{-1}{\beta^2} e^{-\frac{r}{\beta}} \right]_0^\infty$$

$$= \frac{2\mu k_F V_0}{\hbar^2} \times \frac{1}{\beta^2}$$

$$= \frac{2\mu k_F V_0}{\hbar^2 \beta^2}$$

$$\delta_0(k_F) = \frac{\mu k_F V_0}{\hbar^2 \beta^2} \tag{3.35}$$

The well-depth parameters S for Yukawa potential is

$$S = 0.173 \frac{M V_0}{\hbar^2 \beta^2} \tag{3.36}$$

Here M is the mass of the two nucleons interacting, $S \cong 1$ at bound state of the nuclear matter, V_0 is given as,

$$V_0 = 5.784\frac{\hbar^2\beta^2}{M} \quad (3.37)$$

Imputing Eq. (3.37) in Eq. (3.35) will give,

$$\delta_0(k_F) = 2.7834\frac{\mu k_F}{M} \quad (3.38)$$

In $^1S_0$ ground state, the reduced mass μ

$$\mu = \frac{m_p.m_n}{m_p+m_n} \quad (3.39)$$

m_p, is the mass of proton equivalent to 938.26 Mev, and m_n the neutron mass equivalent to 939.56 Mev.

Substituting the values for the masse in Eq. (3.39) the mass reduced $\mu = 469.47\ Mev$. The average mass M for the proton and neutron can be given as the equation,

$$M = \frac{m_p+m_n}{2} \quad (3.40)$$

Putting the value of S the mass M and μ Eq. (3.38) we get,

$$\delta_0(k_F) = 1.1462k_F \quad (3.41)$$

For different value of k_F ranging between 0.0 fm^{-1}, $0.2fm^{-1}$, $0.3fm^{-1}$……$1.6fm^{-1}$, variation of phase shift $\delta_0(k_F)$, with the Fermi momentum k_F were plotted.

The pairing energy gap (Dean & Jensen 2003) for small values of $k_F|a_{nn}|$ is

$$\Delta(k_F) = \frac{8}{e^2}\lambda \quad exp\left(\frac{-\pi}{2k_F|a_{nn}|}\right)$$

(3.42)

Where a_0 is the scattering length for $^1S_0$ the channel, $(a_0 = -19.3\ fm), \lambda$ is a constant ≈ 1,at $k_F = 1.36\ fm^{-1}$, saturation density $\rho_0 = 0.17\ fm^{-3}$
Eq. (3.41) gives Fermi momentum

$$k_F = \left(\frac{\delta_0(k_F)}{1.1462}\right)$$

(3.43)

In the $^1S_0$ channel we assume that the scattering length $a_0 = -19.3 fm, e = 2.718$ and $\lambda \approx 1$ then the value of the pairing energy gap, $\Delta(k_F)$ can be written using Eq. (3.42) by substituting the above values, we get

$$\Delta(k_F) = \frac{8}{2.718^2} . exp\left(\frac{-\pi}{2 \times k_F \times -19.3}\right)$$

$$\Delta(k_F) = 1.0827\ exp\left(\frac{-0.08122}{k_F}\right)$$

(3.44)

Using Eq. (3.43) we substitute the value of k_F in Eq. (3.44), which gives

$$\Delta(k_F) = 1.0827\ exp\left(\frac{-0.0930}{\delta_0(k_F)}\right)$$

(3.45)

Working with derived equations and available values of the constants, data can be generated and tabulated. Graphs have been drawn to show how phase shifts $\delta_l\ (k_F)$, changes with the Fermi momentum k_F. Yukawa potential was substituted to the Born-approximation phase shifts,$\delta_0(k_F)$ for scattering from a spherical potential $V(r)$, and to find the values of phase shifts.

The pairing energy gap is calculated from Eq. (3.43) in the same manner other potentials can be used to calculate for instance the potential proposed by (Hassan and

Ramadan, 1978) can be used to calculate how phase shift $\delta_\ell(k_F)$and pairing of energy gap $\Delta(k_F)$ changes with Fermi momentum k_F.in the limit $l = 0$, the phase shift $\delta_\ell(k_F)$ and its variation with Fermi momentum k_F can be calculated.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1. Introduction

In this chapter theoretical derivations in chapter three were used in calculations and results were obtained and represented on tables and graphs. Discussions of the results are done after each graph and the tables of data are presented in the appendix.

4.2. Results of pairing energy gap and Fermi momentum for $\mathbf{a_{nn} < 0}$

The relationship between the scattering length a_{nn} and the pairing gap $\Delta,$ is given by Eq. (3.14).The constant values in Eq. (3.14) are given in Appendix I, and these values were used to generate data in Eq. (3.15) for the correlation between a_{nn} and Δ.. The data obtained are presented in Table A1 in appendix V.

Figure 4.1 presents graphical results of the correlation of a_{nn} and Δ with Fermi momentum are represented

Solutions from (Eq. 3.14) in (Appendix I) were substituted in Eq. (3.15) to obtain the results for the pairing energy gap. Calculations have been done and values are presented in Table A4.

The results in Table A2 show that the scattering length is negative at $k_F = 0.2\, fm^{-1}$ to $k_F = 1.8\, fm^{-1}$, which means the force is attractive (Changizi 2015) and repulsive for $k_F > 2.0\, fm^{-1}$. At $k_F = 1.4\, fm^{-1}$ it corresponds to $a_{nn} = -16.8607\, fm.$ These results have been used show the variation of the paring energy gap Δ with Fermi momentum k_F (Dean & Jensen 2003)

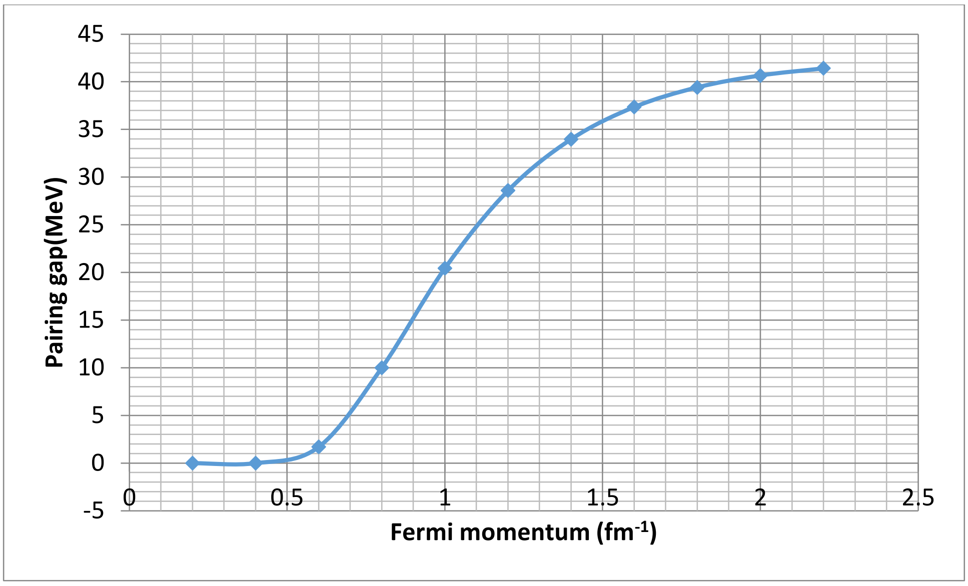

Figure 4.1: Variation of the pairing energy gap against Fermi momentum for scattering of length for $\mathbf{a_{nn} < 0}$

The results obtained in Figure 4.1, shows that pairing energy gap energy increases gradually with Fermi momentum. At $k_F = 0.2fm^{-1}$ pairing energy gap $\Delta = 5.6x10^{-73}\ MeV$ which means it is almost zero, this means the pairing energy is small as $k_F \to 0$ and it increases exponentially at $k_F > 0.4\ fm^{-1}$

4.3. Results of pairing energy gap and Fermi momentum for $\mathbf{a_{nn} > 0}$

The calculations in Table A3 shows that, the pairing energy gap gives negative value. At $k_F = 0.2\ fm^{-1}$ which corresponds to pairing energy gap $\Delta = -5.1x10^{-41}\ Mev$ but on reaching $k_F = 2.0\ fm^{-1}$ energy gap gives a positive value of $\Delta > +3.06x10^{-43}\ Mev$ this means an increases in pairing energy gap. At $k_F = 1.4\ fm^{-1}$ which corresponds to the scattering length $a_{nn} = -16.8607fm$.

Figure 4.2 shows the variation of pairing energy gap with Fermi momentum for scattering length $a_{nn} > 0$. This graph shows that Δ varies exponentially with Fermi momentum, between $k_F = 0.6\ fm^{-1}$ to about $1.0\ fm^{-1}$ and then it is almost constant for $k_F < 0.6\ fm^{-1}$, Δ approaches zero at $k_F = 1.0\ fm^{-1}$.

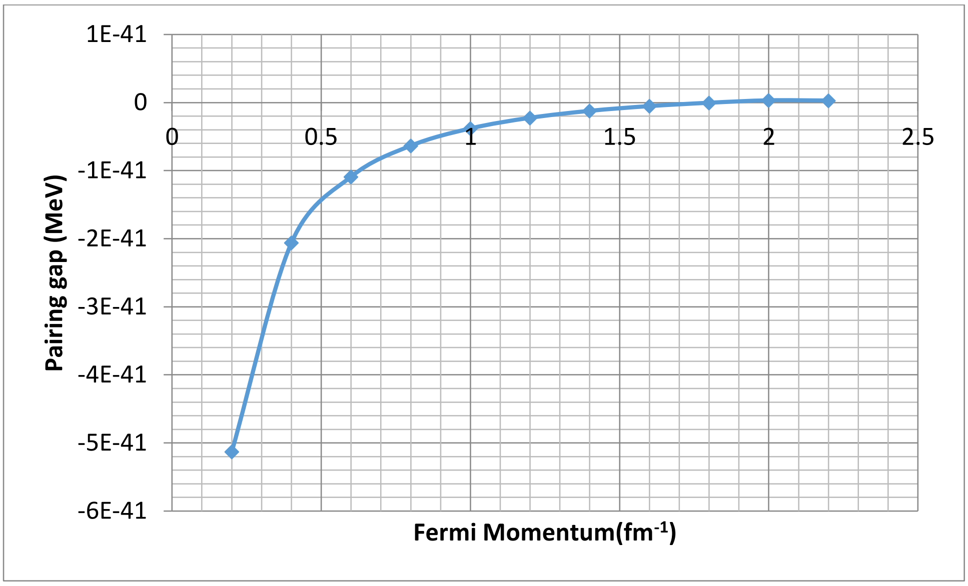

Figure 4.2: Variation of pairing energy gap with Fermi momentum for scattering length $a_{nn} > 0$.

4.4. Calculations on pairing Energy gap and Fermi momentum using Yukawa potential

Eq. (3.44) have been used to calculate the values for pairing using Yukawa interaction potential to investigate its effects on the pairing energy gap. These effects also has influence on phase shift which means the will be some difference from results obtained compare to Eq. (3.15) and Eq. (3.16)

The results in Table A4 gives positive values for pairing energy gap. At $k_F = 1.4\ fm^{-1}$ pairing energy gap $\Delta = +0.9473\ MeV$ (Gezerlis, *et. al.*, 2014). This corresponds to the repulsive effect on the interactions of the paired nucleons. Fig. 4.3 shows the variation of pairing energy gap Δ with Fermi momentum, k_F. This results shows that the pairing energy varies exponentially with the Fermi momentum. The value of Δ increases gradually with k_F, and for $k_F = 1.0\ fm^{-1}$, it has an approximate value of 0.89 MeV (Dortmans and Amos 2001)

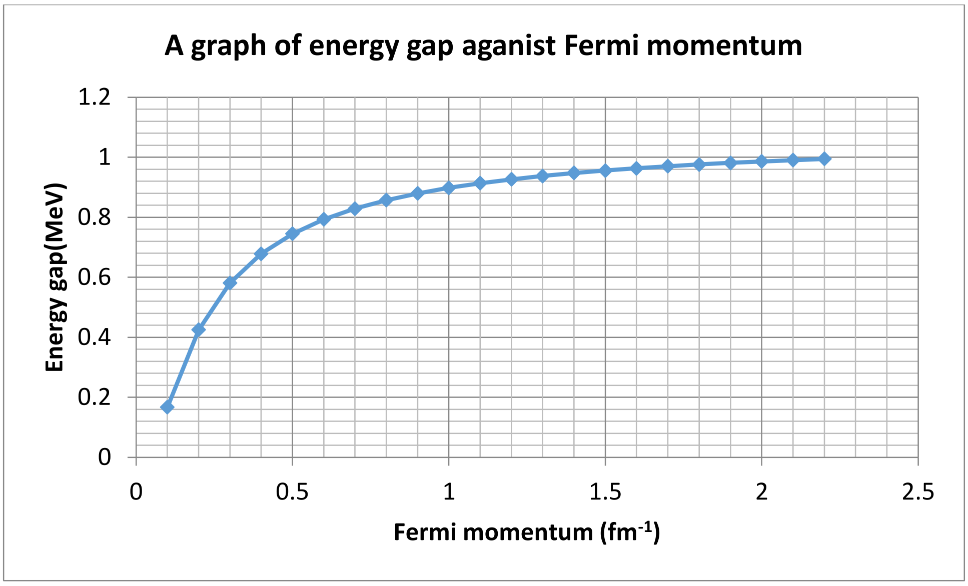

Figure 4.3: Variation of pairing Energy gap with Fermi momentum

4.5. Relation between the Phase shift and Fermi momentum using

Using Eq. (3.41), and $\delta_0(k_F) = 1.1462k_F$ to calculate the values of phase shift, $\delta_0(k_F)$ for the values of k_F ranging between $0.0 - 1.6fm^{-1}$ … … are calculated and a graph of phase shift $\delta_0(k_F)$, against Fermi momentum k_F have been plotted (Gezerlis, et. al., 2014). Results in Table A.4 shows that values of Fermi momentum is directly proportional to the phase shift using Yukawa interaction potential. Fig. 4.4 shows that the phase shift δ_l increases linearly as Fermi momentum k_F increases, and it is zero in the ground state, since $k \rightarrow 0$.

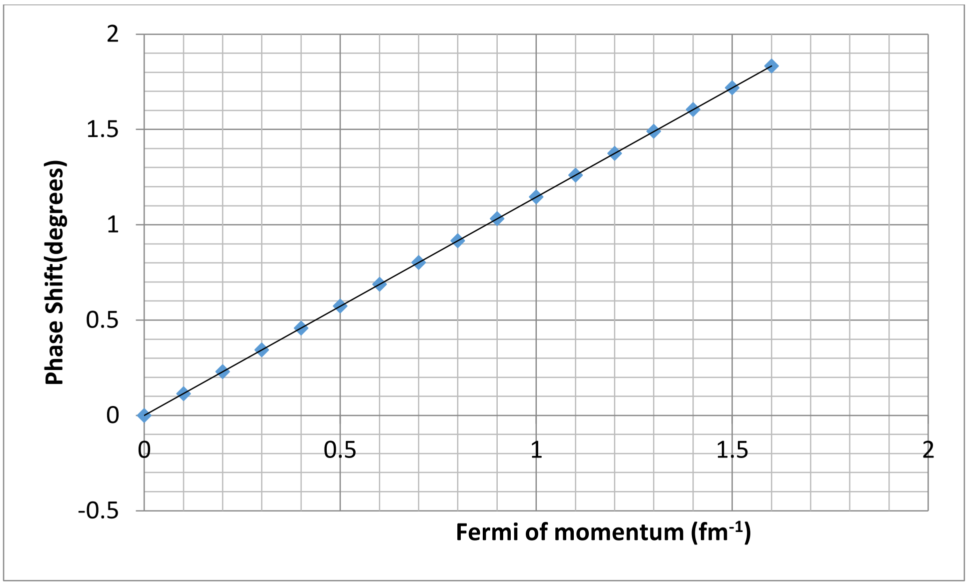

Figure 4.4: variation of phase shifts δ_l with the Fermi momentum k_F

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1. Conclusions

This research concentrated on for major objectives which have been achieved through calculations and representation of results obtained. Microsoft excel have been used to solve equations and to plot graphs. Correlation between nucleon-nucleon interactions and pairing energy gap have been established using scattering length equation. Yukawa interaction potential has significant influence on the Phase shift which corresponds to the Fermi momentum.

The relation between scattering length and pairing energy gap was establish in Eq. (3.42) and solutions calculated and presented in Table A2. Numerical values of pairing energy gap have been obtained in Table A2, which gives pairing energy gap $\Delta = 1.69MeV$ at $k_F = 0.6\, fm^{-1}$ while that of CD-Bonn and Nijmegen At $k_F = 0.6\, fm^{-1}$ is $\Delta = 2.08MeV$. At $k_F = 1.4\, fm^{-1}$ which corresponds to the scattering length $a_{nn} = -16.8607fm$, approximately the same as that of (Dean & Jensen 2003), with scattering length of $a_{nn} = -18.03\, fm$ at $k_F = 1.5\, fm^{-1}$.

In Both Eq (3.15) and Eq (3.14) it gives a negative scattering length which means the force is attractive. Pairing energy gap is different with Eq. (3.15) and Eq. (3.44) having positive values while Eq. (3.16) negative pairing energy gap values. Positive values of pairing energy gap means nucleons were paired together while negative means there was breaking of the nucleon pairs. Correlation between pairing energy gap and Fermi momentum using Yukawa potential have been established in Eq. (3.44).

The relation between pairing gap equation and Phase shift have been determine in Eq.

(3.45) using Yukawa interaction potential. The results shows that Phase shift is directly dependent on Fermi momentum.

5.2. Recommendations

In future the problem can be done to investigate, if there is direct relation between phase shift and pairing energy gap. Elementary particles can also be used, although it requires complex and complicated techniques which will need the use of many-body techniques including Greens functions will be done. Current discoveries on elementary particles leads to the suggestions of various types of nuclear interactions with many parameters involved. Elementary particles in the nucleus are neutrons and protons. But using the theory of the Quantum Chromo-Dynamics (QCD), neutrons and protons are not the only elementary particles. There are nuclear forces among the neutrons which are; neutron- neutron force, proton-proton force and neutron-proton forces which are treated as charge independent particles. Within the Theory of QCD, there is charge symmetry breaking (CSB) and consequently there will be n-n-scattering length, the p-p-scattering length and n-p scattering length changes. Since they scatter there must be a corresponding phase shift changes of $a_{nn}, a_{pp}\ and\ a_{np}$, and these will give different relationships of the nuclear force.

REFERENCES

Bardeen, J., Cooper, L. N. & Schrieffer, J. R. (1957). Theory of superconductivity, *Phys. Rev.*, Vol. 108(5), pp. 1175 – 1204.

Blatt, J.M & Weiss Kopf, V.F. (1982) *Theoretical Nuclear Physics*, John Wiley and Sons, ISBN:

Brown, A.B. (2013). Pairing and shell gap in *Nuclei. Journal of Phys*. Conf. Ser. 445, 012010. New York USA

Chadian, K and Sabatier, P. C. (1992). *Inverse problems in quantum scattering theory,* Springer, New York p. 105.

Changizi S. A. and Qi, C. (2015). Empirical pairing gaps, shell effects and di-neutron spatial correlation in neutron-rich nuclei. *Nuclear physics* A 940 2010.

Cottingham W. N., M. Lacombe, B. Loiseau, J. M. Richard and R (1973). "Nucleon-Nucleon interaction from pion-nucleon phase-shift analysis". Vinhman, *Phys. Rev*. D 8, 800

Dean D.J & Hjorth-Jensen M. (2003). Pairing correlations in finite nuclear systems. *Rev.Mod.Phys*.Vol75607

Dortmans P. J. and K. Amos (2001). *Medium corrections to Nucleon-Nucleon interactions.* Parkville, Victoria Australia, 0352, P. 1-25

E. Elpelbaum, H.-W. Hammer & Ulf-G. Meiner (2009). "Modern theory of nuclear forces", *Rev. Mod. Phys*. 81, 1773

Etrin. V.A (2017). Modified Coulomb Law World Science News (WSN) 07 163-174

Fetter. A.L and Walecka J.D. (2003). *Quantum Theory of Many-Particle Systems*. Dover Publications. Inc. New York page 385

Gezerlis A, Pethick C. J. & Scwenk A, (2014). *Pairing and superfluidity of Nucleons in neutron stars.* Vol 2, P.1

Ghosh AL S.N (2010). *Nuclear Physics*, S. Chand Company Ltd, New Delhi-INDIA

110055

Gross, F., T.D Cohen, E. Epelbaum & R. Machleidt (2011) “Conference discussion of the nuclear force”, *Few-body Syst*.50, 31 1110:376

Hamada, T and I. D. Johnston (1992) “A potential model representation of two-nucleon data below 315 MeV”, *Nucl. Phys*. 34, 382

Hassan M.Y.M & Ramadan S. (1978). On the properties of nuclear matter with an excess of neutrons, spin-up neutrons and spin-up protons using effective nucleon-nucleon potential, *acta physica polonica*.B9.;989.

Heiselberg H. & Hjorth-Jensen M. (2000). Phases of Dense matter in neutron stars. *Phys.Rev*.B28237

Khanna K.M. (1962). On super fluidity and Phase transition in atomic nucleus. Prog.Theor.*Physics.Japan*.vol.28 205

Khanna K.M. (1968). Alpha particle model for nuclear matter, J. *phy. Soc. Japan*, 23 1429

Kodel. V.A & Clark. J.W, (1996). Nuclear Physics, *Nuc. Phys.* A, 598, P390

Lacombe M., B. Loiseau & J. M. Richard. (1973) “Nucleon-Nucleon interaction from pion-nucleon phase-shift analysis”. Vihaan, *Phys. Rev.* D 8, 800.

Lamb F.K & D.L. Lambert (1991*) Frontiers of Stellar Evolution Astronomical Society of the Pacific San Francisco*. P.299

Machleidt, R, Q. MacPherson, Marji, R. Winzer, Ch. Zeoli and D.R. Entem, (2015) “*Recent progress in the theory of nuclear forces*” arXiv:1210.0992nucl-th

Preston, M.A. (1962). *Physics of nucleus*, Addison-Wesley Publishing Company Mass, U.S.A

Reid R. V. (1968) “Local phenomenological nucleon-nucleon potentials”, *Ann. Phys.(*NY) 50, 411

Schiff. L.I, (1968) *Quantum Mechanics*, McGraw Hill Book Company, New York

Shapiro. S.L & Tenkolsky. S.A (1983). *The Physics of compact objects*. Wiley, New York, P.188

Yukawa H. (1985). "On the interaction of elementary particles", Proc. *Phys. Math. Soc.* Jn. 17, 48.

APPENDICES

Appendix I

Table A1: Coupling strength and density gradient term using Lead isotopes

$f_{\mathrm{ex}}^{\varepsilon}$	h^{ε}	f_{∇}^{ε}
-0.56	0	0
-1.20	0.56	2.4
-1.60	1.10	2.0
-1.79	1.36	2.0
-2.00	1.62	2.0
-2.40	2.16	2.0

Table A2: Calculation of scattering length and pairing energy gap using Eq. (3.15)

f_{ex}^{ε}	kf	π	f_{cr}^{ε}	a_{nn}	c	ϵ_F	Δ
-0.5	0.2	3.142	-1.912	-0.10637	1.083	36.57	5.6E-73
-1.1	0.4	3.142	-1.912	-0.81382	1.083	36.57	0.000591
-1.3	0.6	3.142	-1.912	-1.91416	1.083	36.57	1.697753
-1.4	0.8	3.142	-1.912	-3.28535	1.083	36.57	10.00057
-1.5	1	3.142	-1.912	-5.468	1.083	36.57	20.43832
-1.6	1.2	3.142	-1.912	-9.24231	1.083	36.57	28.58276
-1.7	1.4	3.142	-1.912	-16.8607	1.083	36.57	33.97814
-1.8	1.6	3.142	-1.912	-38.6197	1.083	36.57	37.35332
-1.9	1.8	3.142	-1.912	-428.035	1.083	36.57	39.4198
-2	2	3.142	-1.912	68.26709	1.083	36.57	40.66864
-2.1	2.2	3.142	-1.912	36.9078	1.083	36.57	41.40963

Table A3: Calculations of pairing energy gap and Fermi momentum for using Eq. (3.16)

k	π	f_{cr}^{s}	a_{nn}	ħ^2	m	2ρ	Δ
0.2	3.142	-1.912	-0.40691	1.11E-68	1.67E-27	0.1582	-5.1E-41
0.4	3.142	-1.912	-1.0125	1.11E-68	1.67E-27	0.1582	-2.1E-41
0.6	3.142	-1.912	-1.91416	1.11E-68	1.67E-27	0.1582	-1.1E-41
0.8	3.142	-1.912	-3.28535	1.11E-68	1.67E-27	0.1582	-6.4E-42
1	3.142	-1.912	-5.468	1.11E-68	1.67E-27	0.1582	-3.8E-42
1.2	3.142	-1.912	-9.24231	1.11E-68	1.67E-27	0.1582	-2.3E-42
1.4	3.142	-1.912	-16.8607	1.11E-68	1.67E-27	0.1582	-1.2E-42
1.6	3.142	-1.912	-38.6197	1.11E-68	1.67E-27	0.1582	-5.4E-43
1.8	3.142	-1.912	-428.035	1.11E-68	1.67E-27	0.1582	-4.9E-44
2	3.142	-1.912	68.26709	1.11E-68	1.67E-27	0.1582	3.06E-43
2.2	3.142	-1.912	75.0938	1.11E-68	1.67E-27	0.1582	2.78E-43

Table A4: Relation between Pairing energy gap and Fermi momentum using Yukawa in Eq. (3.44)

k	8/e^2	Δ
0.1	1.0827	0.166843
0.2	1.0827	0.425019
0.3	1.0827	0.580463
0.4	1.0827	0.678357
0.5	1.0827	0.744849
0.6	1.0827	0.79276
0.7	1.0827	0.828857
0.8	1.0827	0.857005
0.9	1.0827	0.879556
1	1.0827	0.898024
1.1	1.0827	0.913423
1.2	1.0827	0.926456
1.3	1.0827	0.937629
1.4	1.0827	0.947314
1.5	1.0827	0.955788
1.6	1.0827	0.963265
1.8	1.0827	0.975856
2	1.0827	0.986048
2.2	1.0827	0.994466

Table A5: results for Phase shift and Fermi momentum using Yukawa potential in Eq. (3.41)

k (f/m)	phase shift (°)
0	0
0.1	0.11461
0.2	0.22922
0.3	0.34383
0.4	0.45844
0.5	0.57305
0.6	0.68766
0.7	0.80227
0.8	0.91688
0.9	1.03149
1	1.1461
1.1	1.26071
1.2	1.37532
1.3	1.48993
1.4	1.60454
1.5	1.71915
1.6	1.83376

Table A6: Constant Values and Symbols

Quantity	Symbol	Value
Speed of light in vacuum	c	$2.99792458 \times 10^{8} \mathrm{m/s}$
Gravitational constant	G	$6.67259\ x\ 10^{-11} m^{3} /(kgs^{2})$
Electron charge, elementary charge	e, e_o	$1.60217733 \times 10^{-19} \mathrm{C}$
Permittivity constant of free space or Electric constant	$\in_o = (\mu_o c^2)^{-1}$	$8.854187817 \times 10^{-12} F/m$
Permeability of free space or magnetic constant	μ_o	$12.566370614 \times 10^{-7} N/A^2$
Planck's constant in eV s	h	$6.6260755 \times 10^{-34} J.s$ $4.135\ 66743 \times 10^{-15}\ eV.s$
Planck's constant in eV s	$\hbar = \frac{h}{2\pi}$	$1.05457266 \times 10^{-34} J.s$ $6.58211915 \times 10^{-16}\ eV.s$
Avogadro's number	N_A	$6.0221367 \times 10^{23} mol^{-1}$
Faraday constant	$F = N_A e_o$	$9.6485309 \times 10^{4}\ C/mol$
Electron mass in atomic mass unit energy equivalent	m_e	$9.1093897 \times 10^{-31}$ $5.4857990945 \times 10^{-4}\ u$ $0.51099906\ MeV$
Rydberg constant	$R_\infty = (2h)^{-1} m_e c \propto^2$	$1.0973731534 \times 10^{7} m^{-1}$
Fine-structure constant	$\propto = e_o^2 (2e_o hc)^{-1}$ $\propto^{-1}$ $1/4\pi e_o$	$7.29735308 \times 10^{-3}$ 137.0359895 $8.99 \times 10^{9}\ N.m^2/C^2$
Electron radius	$r_e = \hbar (m_e c)^{-1} \propto$	$2.81794092 \times 10^{-15}\ m$
e^- Compton wavelength	$\lambda_C = h(m_e c)^{-1}$	$2.42631058 \times 10^{-12}\ m$
Bohr radius	$a_o = r_e \propto^{-2}$	$5.29177249 \times 10^{-11}\ m$
atomic mass unit	$u = 1/12\, m(^{12}C)$	$1.6605402 \times 10^{-27}\ kg$
proton mass	m_p	$1.6726231 \times 10^{-27} kg$

in atomic mass unit energy equivalent		$1.00727646688\ u$ $938.27231\ MeV$
neutron mass in atomic mass unit energy equivalent	m_n	$1.6749286\ \times\ 10^{-27}\ kg$ $1{:}\,00866491560\ u$ $939.56563\ MeV$
magnetic flux quantum	$\phi_o = h(2e_o)^{-1}$	$2.06783461\ \times\ 10^{-15}\ Wb$
specific electron charge	$-e_o {m_e}^{-1}$	-1.75881962 $\times\ 10^{11}\ C$ $/kg$
magnetic moment of electron	μ_e	$9.2847701\ \times\ 10^{-24}\ J/T$
Bohr magneto	$\mu_B = e_o \hbar(2m_e) - 1$	$9.2740154\ \times\ 10^{-24}\ J/T$
nuclear magneto	$\mu_N = e_o \hbar\left(2m_p\right)^{-1}$	$5.0507866\ \times\ 10^{-27}\ J/T$
magnetic moment of proton	μ_p	$2.792847351\ \mu_N$ $1.41060761\ \times 10^{-26}\ J/T$
deuteron mass energy equivalent	m_d	$2.01355321270\ u$ $1875.61282\ MeV$
magnetic moment of neutron	μ_n	$-1.91304273\ \mu_N$ -0.96623645 $\times\ 10^{-26}\ J/T$
magnetic moment of deuteron	μ_d	$0.8574382329\ \mu_N$ $0.433073482\ \times 10^{-26}\ J/T$
gyromagnetic ratio	γ_p	$2.67522128\ \times\ 10^{8}\ rad$ $/sT$
quantum Hall resistance	R_H	$25812.8056\ \Omega$
universal gas constant	R	$8.314510\ J/(mol\ K)$
Boltzmann constant	$k, k_B = R{N_A}^{-1}$	$1.380658\ \times\ 10^{-23}\ J/K$
Stefan-Boltzmann constant	σ $= \pi^2 {k_b}^4 (60\hbar^3 c^2)^{-1}$	$5.67051\ \times\ 10^{-8}\ W/m^2K^4$
Wien's constant	$b = \lambda_{max} T$	$2.897756\ \times\ 10^{-3}\ m.K$

Appendix II : Similarity Report

Document Viewer

Turnitin Originality Report

Processed on: 08-Jun-2021 12:31 EAT
ID: 1602753354
Word Count: 9367
Submitted: 1

SC/PGP/003/2011 By Koech Kiprono Willy

Similarity Index	Similarity by Source	
14%	Internet Sources:	9%
	Publications:	8%
	Student Papers:	2%

include quoted   include bibliography   excluding matches < 5 words   mode:
quickview (classic) report   Change mode   print   refresh   download

1% match (Internet from 07-May-2016)
http://www.rroij.com

1% match (publications)
Ø. Elgarøy. "Nucleon-nucleon phase shifts and pairing in neutron matter and nuclear matter", Physical Review C, 03/1998

1% match (student papers from 03-Dec-2020)
Submitted to University of Witwatersrand on 2020-12-03

1% match (Internet from 24-Jul-2020)
https://mafiadoc.com/stellar-superfluids_5cb53c43097c474b628b4583.html

1% match (Internet from 13-Jan-2010)
http://www.phy.uct.ac.za

<1% match (Internet from 19-Aug-2019)
http://www.rroij.com

<1% match (Internet from 29-Mar-2021)
https://archive.org/stream/in.ernet.dli.2015.177153/2015.177153.Lectures-In-Theoretical-Physics-Vol-1_djvu.txt

<1% match (Internet from 10-Jun-2020)
http://archive.org

<1% match (Internet from 30-Dec-2020)
https://archive.org/stream/in.ernet.dli.2015.459916/2015.459916.Nuclear-Physics_djvu.txt