MATHEMATICAL MODELLING OF FLOW OF FERTILIZER-WATER MIXTURE THROUGH SOIL AND ITS EFFECT ON PLANT GROWTH.

BY

RUTTO KIPKORIR TIMOTHY

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DECLARATION

Declaration by student

This work is my original work and has not been submitted for a degree in any other University; and shall not be reproduced in part or full, or in any format without prior written permission from the author and/or University of Eldoret.

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RUTTO KIPKORIR TIMOTHY

SSCI/MAT/M/008/17

Declaration by supervisors

This thesis has been submitted with our approval as University Supervisors.

Signature…………………… Date……………

DR. MAREMWA SHICHIKHA

Department of Mathematics and computer science

University of Eldoret, Kenya

Signature…………………….. .Date………….

DR. KANDIE JOSEPH

Department of Mathematics and computer science

University of Eldoret , Kenya

DEDICATION

To my late father; Nicholas Cherono and mother; Josphina Kandie who supported me throughout my education, my brother Edwin Rutto who motivated me to develop interest in mathematics, my beloved wife, Faith and my two children; Travin and Fabriana who have been very understanding during my studies; you will forever remain special to me.

My university lecturer Dr.G.O. Lawi who made me develop interest in the field of applied Mathematics, my other siblings; Jonathan, Abel, Prisca, Linet, Lydia and Hillary for their encouragements and support in research.

ABSTRACT

The demand for food in developing countries is increasing daily due to population growth and dropping crop yields caused by climatic changes and acidic soils due to continuous use of inorganic fertilizers that are meant to boost soil fertility. This study aims to demonstrate and explore how mathematical modelling can be used to aid people gain knowledge and understanding of plants and, in particular, interactions between plants, fertilizers, soil, and water. The primary objective is to convince members of the agricultural, mathematical and biological cultures of the need to work together in establishing mechanistic simulations with quantitative data to help them understand flow of soil solutions and their effect on plant growth. The mathematical models used are based on the fundamental knowledge of fluid flows and plant growth that is necessitated by nutrient absorption from the soil by plant roots. Such models allow for a deeper understanding of plant science at the most basic level and can aid us in dealing with real-world issues such as food scarcity, soil pollution and global warming in the world, Kenya as a target country. We used mathematical model equations which have been made to describe fertilizer as well as soil water flow, their concentrations, uptake by a plant root system and plant growth then solved the equations by finite difference methods with the help of MATLAB program. The technique of explicit finite difference was used to solve the governing equations with the help of analytical descriptions. The results indicate that as time of simulation increases, the concentration of fertilizer also increases thus increasing the growth factor which in turn affects the length of plant growth.

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LIST OF ABBREVIATIONS

ADE: Advection Dispersion Equation

GM: Growth Mitosis

ppm: parts per million

REV: Representative Elementary Volume

SWAP: Soil-Water-Atmosphere-Plant

WOFOST: World Food Studies

LIST OF NOTATIONS

∇^2: Laplacian operator symbol.

C: the concentration of fertilizer solution in soil (ppm)

Δt: change in time (s)

C_n: Concentration of nutrients (ppm)

d: diffusion coefficient (m/s)

D_L: longitudinal dispersion coefficient (cm^2/s)

h: pressure head (m)

k: hydraulic conductivity in x direction

$\underline{k}$: permeability

K: unsaturated hydraulic conductivity tensor (ms^{-1})

L (t): plant size (length/height) that depends on time (t)

n: soil porosity

P: pressure

R: growth mitosis factor responsible for growth of plants

S: the sink term for root water extraction from soil ($L^3L^{-3}\ T^{-1}$)

T: Retardation factor

t: time in seconds (s)

V: seepage velocity (m/s)

x: space co-ordinate (m)

z: vertical space co-ordinate (m).

θ: volumetric water content (Dimensionless)

μ : fluid viscosity

ρ: density of fluid ($Kgcm^{-3}$)

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CHAPTER ONE

INTRODUCTION

1.1 Background Information

The most basic principles relating to plant growth are the movement of fluids in the soil and the absorption of nutrients by plant roots. Numerous studies have been published using the Richards (1931) equation to resolve a variety of problems involving the flow of water in the unsaturated zone. For a steady-state variation, the dynamic water flow equation is a Darcy expression and a formulation of mass balance. Milly (1988) published a systematic analysis of unsaturated soil water flow modeling, while Marino and Tracy (1988) published a comprehensive review of modeling of root-water absorption in conjunction with the solution of the Richard's equation. In our research, we create a mathematical model to understand how fertilizer-water mixtures flow in soil and how their rates influence plant growth and development. The knowledge of transport of contaminants through soil in ground water is used in this research together with equations describing plant growth caused by the concentration of inorganic fertilizer around the root surface. The model is used to calculate the amount of fertilizer consumed by plant roots in relation to the plant's actual growth size in length or volume.

In areas with low soil fertility, large quantities of fertilizer are used, whereas in areas with higher soil fertility, small amounts of fertilizer are used to increase soil fertility. The government has no specific regulations on the amount of fertilizer to be applied hence no control over the hazards that may cause on plants as a result of excessive use of fertilizer in soil (Act No.20 of 2015). In recent years, there has been an unpredictable change in seasons of rainfall and thus pausing danger on food security in our country Kenya. One of the factors that may lead to seasonal rainfall is due to continuous use of soluble chemicals present in fertilizer which end up affecting plant populations and possibly causing the extinction of certain less adapted species. This research is based on the fact that in Uasin Gishu County, Kenya 'maize and wheat are major crops grown in the area'. Maize crops thrive well in Loamy soil and in less sloppy land having enough Rainfall supply. Our work presents a mathematical model that would be useful in monitoring the rate of flow of fertilizer-water mixture and its concentration in soil and on the root surface, therefore help control amount of fertilizer as well as growth of plants.

1.2 Basic Concepts

1.2.1 Governing Equations

The continuity equation, which expresses mass conservation, is generally required for the hydrodynamic definition of a fluid-flow problem and a relationship between states among them: temperature, stress, and density. The problem with the flow is described mathematically by a system of partial differential equations, which can be more or less complex, the solution of which necessitates the Boundary conditions and, if the flow is unsteady, initial conditions describing the particular flow situation are specified.

The movement of fluids inside the soil can be studied on a minuscule scale by taking the soil into account as a dispersed system and, even if only conceptually, solving the problem using the Navier-Stokes's equations. The Navier-Stokes's equation written in Cartesian co-ordinates is;

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right] = -\frac{\partial P}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \text{.................}(1.1)$$

Flow velocities in real time have significant magnitude and direction differences due to the complexities of the paths taken by single fluid particles as they pass through the pores that are intertwined in soil, Consequently, a careful examination of the definition of flow pattern at any point in the domain is virtually impossible. The understanding of flux density, on the other hand, is of greater importance in many applications. As a result, processes of soil movement and transport are usually defined at a macroscopic level by specifying a Representative Elementary Volume (REV) and a collection of averaged values and balance equations.

We first make an assumption that the fertilizer is soluble in water and the solution is a homogeneous mixture. The fertilizer is rich in nutrients required by plants during growth and development. The velocity which the fertilizer solution flows in soil has direct effects on uptake by plant root system hence affecting plant growth.

In our research, the model considers the equation that regulates vertical, unsaturated – convective or isotropic –soil water flow and contaminant transport. We use the following equation for transport of contaminants through soil in groundwater, Negm et., al. (2016):

$$D_L \frac{\partial^2 C}{\partial x^2} - V\frac{\partial C}{\partial x} = T\frac{\partial C}{\partial t} \text{..}(1.2)$$

The hydraulic properties of the studied soil system are used to simulate water flow and solute transport through soil medium. Direct measurements and tests on soil hydraulic properties can be performed in the laboratory or on the ground through experimentation However, since these approaches can be expensive and take a long time, different methods for estimating soil hydraulic properties have arisen. These properties can be estimated using soil knowledge that is readily obtainable and simple to calculate. Functions in mathematics for estimating soil hydraulic variables using simple soil data for example soil texture, the amount of organic matter, and densities of bulk are applied in this research.

1.2.2 Solute transport modeling

Below are a few of the most important processes which influence transport of solutes in soil. **Advection** is the average rate of solute mass transportation caused by the flow of water.

Advection can be represented by the Darcy's equation as:

$$Q = KA\,\frac{\emptyset_1 - \emptyset_2}{\Delta s} \quad (1.3)$$

If only advection regulated solute transport in the soil, the transport velocity will be the same as the average water flux.

Hydrodynamic dispersion: This process involves a solute spreading through mechanical dispersion and diffusion of molecules in the direction of the flow (longitudinal dispersion) and parallel to the ground (perpendicular or transverse dispersion). The soil undergoes mechanical dispersion resulting from variations in pore size, differences in the length of the flow path, continuous mixing between pores (because of the configuration of the soil's pores), and differences in velocity of pore transport. On both a micro (within pores) and macro (outside pores) scales (preferential flow through soil cracks), this happens in soil medium. Solutes are transferred through molecular diffusion as a result of a concentration gradient.

Dispersion by mechanical means and diffusion of molecules are two main processes that cause hydrodynamic dispersion. Constituents pass from a high-concentration area to a low-concentration area in this process; the greater concentration difference, the faster the diffusion rate.

The equation for advection-dispersion (ADE): The most widely used mathematical description for transport of solutes through soil is the Advection Dispersion Equation (ADE). The value of the dispersion coefficient is needed for numerical or analytical ADE solutions. Field or laboratory experiments may be used to measure the hydrodynamic dispersion coefficient. Therefore, ADE is written mathematically according to Moene and Van Dam (2013) as:

$$\frac{\partial(\theta.C)}{\partial t} = \frac{\partial}{\partial z}\left[D_{sh}(V,\theta)\frac{\partial C}{\partial z}\right] - \frac{\partial(q.c)}{\partial z} - S_s \quad (1.4)$$

1.2.3 Plant Growth Model Equations

Consider L (t) as a plant size that varies over time and is larger compared to the circumference of the trunk; As a result, the range of values of x; $0 \leq x \leq L(t)$ is used. The left end point x = 0 is equivalent to the root, whose key function is to enable flux of nutrients from the fertilizer, as determined by the boundary condition. The boundary on the left-hand side is set (fixed), and the right endpoint x= L(t) relates to the top growing stem of the plant. The value L(t) increases over a period of time, t. The rate of growth is therefore determined by the concentration of metabolites (nutrients present in fertilizer solution) at the point x = L(t) represented by the symbol, R. This is mathematically expressed as;

$$\frac{dL}{dt} = f(R) \quad (1.5)$$

The interval $0 \leq x \leq L(t)$ represent the cells that have been divided which transport nutrients absorbed on the plant's root surface to its top part of the stem(apex). The nutrients are fertilizer-water solutions whose concentration is denoted by C_n, which is a function of the variables x, q and t. The diffusion equation below with convective terms is used to explain how it moves in space according to Bessonov et., al. (2000):

$$\frac{\partial Cn}{\partial t} + v_x\frac{\partial Cn}{\partial x} + v_y\frac{\partial Cn}{\partial x} = d\Delta C_n \quad (1.6)$$

Where $v=(v_x,v_y)$ as Darcy rule stipulates i.e

$$v_x = -K\frac{\partial P}{\partial x},\ v_y = -K\frac{\partial P}{\partial y} \quad (1.7)$$

$K=\frac{þ}{\mu}$ where permeability is þ and viscosity is μ, pressure is P, and the diffusion coefficient is d.

Consider the fertilizer solution (fluid) to be incompressible and that it evenly fills the xylem of a plant (the portion of the plant tissue responsible for nutrient transport from the roots to the apex and a layer of cambium that is found within the cambium) we get:

$$u = \frac{dL}{dt} \quad (1.8)$$

We compute the equation $\frac{\partial Cn}{\partial t} + u\frac{\partial Cn}{\partial x} = d\frac{\partial^2 Cn}{\partial x^2}$ by setting initial and final boundary conditions $x=0$: $C_n = 1$, $x = L(t)$: $d\frac{\partial Cn}{\partial x} = -g(R)C_n$ (1.9)

The final boundary condition demonstrates that nutrient flux through the plant's main body to the meristem has a proportional relationship with the concentration $C_n(L,t)$. The factor g(R) indicates that proliferating cells will control the flux. The equation describing evolution of R with the width of the meristem, h is given by

$$h\frac{dR}{dt} = g(R)Cn - \sigma R \qquad (1.10)$$

, where g(R) describes the production of growth mitosis (GM) factor, R in the meristem while $-\sigma R$ is equivalent to the amount consumed by the plant. These equations are a generic plant growth model based on a medium that is continuous and biological theories that plant growth is regulated by a chemical substance denoted R, referred as plant growth mitosis (GM) factor.

1.2.4 Boundary and Initial Conditions

In any flow domain, a set of conditions must be met before the flow equations can be solved; that act at the start of flow and at the domain boundary. These conditions are necessary to provide a solution to a set of equations and simplify them. Cauchy initial and boundary conditions are used in this research.

1.2.5 Basic Assumptions

- The amount of light is enough for plant growth and is uniformly distributed.
- The type of fertilizer chosen is soluble in water.
- The slope of land where the crops are grown has limited effect on fertilizer flow through soil.
- Other environmental factors such as soil PH, atmospheric air, humidity, soil texture and temperature have little effect.
- The type of soil good for crop growth is Loam soil.

- Soil parameters can be obtained theoretically and others measured experimentally.
- Photosynthesis is ignored.
- The concentration of nutrients in soil and on the root surface determines the rate of growth.
- Root growth is ignored and growth measured from ground surface upwards.
- Production of growth mitosis factor, R is autocatalytic.

1.3 Statement of the Problem

Numerous studies have been done in the past on flow of water and solutes through soil medium. However, combining or relating with plant growth has been constrained in all cases. As a result, we have included integration of fertilizer-water movement through soil with nutrient uptake and a link to plant growth in this study. We use some of the existing equations for solute flow through soil and plant growth models, but our conclusions and explanations are based on how these flows affect nutrient concentration from fertilizer and, as a result, plant growth.

1.4 Objectives

1.4.1 General objective

The primary goal of this study is to figure out the availability of nutrient components in fertilizer solutions in terms of time and space, as well as to assess the effects of their flow through soil on the plant growth.

1.4.2 Specific Objectives

i. To use mathematical modelling equations to determine the flow of a fertilizer-water mixture through the Rhizosphere.

ii. To formulate and solve analytically the equations involved in the modelling and relate the concentration of fertilizer and growth factors with time of flow.

iii. To make use of the solutions to investigate the effect of flow of the fertilizer-water mixture through soil on apical plant growth.

1.5 Significance of the study

The study finds its application in agricultural and biological engineering sector because it focuses on soil-water-nutrient and plant interaction. The research can be employed to optimize crop production by regulating amount of flow of fertilizer thus improving food supply and help in the fight against hunger. Nutrients supplied by fertilizer applied dissolves in water and gets absorbed by the plant root system in the rhizosphere. However, since the velocity of flow across the root surface influences the absorption of nutrients, not all nutrients are consumed by the roots; this causes the remaining unabsorbed chemicals to cause soil pollution hence endangering microorganisms in the soil and some loss of soil fertility.

CHAPTER TWO

LITERATURE REVIEW

The soil-plant atmosphere continuum encompasses plant and soil processes as well as their interactions. The environment and climate have gotten a lot of coverage in the soil-plant-atmosphere continuum, compared to the processes taking place beneath the surface that will ultimately result in studies that involve interactions between plants and soil, Feddes, et.al., (2001); Wang and Smith (2004).

The area of soil surrounding the root of a plant has been dubbed the rhizosphere (Hiltner (1904); Tinker and Nye (2000); Darrah, et.al., (2006)), rhizosphere models are proof of concepts that deal with this field. These models enable researchers to assess the impact of nutrient uptake and plant growth feedback loops on growth and development of the crop as a whole. Fertilizers can be added to the ground and recycled by soil–crop systems, where they become an important source of nutrients while also improving the physical structure of the soil (Egrinya, et.al., (2001); Larney, et.al., (2000); Marinari, et.al., (2000)).

Since they regulate nutrient transport and indirectly support hormone transport through root uptake, processes involving soil water and the root zone are critical for plant health and overall growth (Ehlers and Goss, 2003). As a result, the flow of soil fertilizer solution and vegetation production, or rather plant growth, are inextricably connected. Crop growth and production are influenced by the total amount of nutrients available to the plant.

Understanding the rate at which nutrient components bind to one another, pass through, and leave the soil is important for quantifying nutrient availability (Havlin, et.al., 2013). The rhizosphere requires the transport of nutrients from the bulk soil toward the root surface to meet crop nutrient demand (Marshner and Marshner, 1995). The simplest single root uptake model uses Michaelis-Menten kinetics to account for nutrient transport in the soil through convection and diffusion, as well as nutrient desorption from the solid phase of the soil and uptake at the surface of the root (Darrah, et.al., 2006).

Modeling the simultaneous water and nutrient absorption by plant roots from partially saturated soils was the focus of Roose and Fowler (2004). Roose and Fowler (2004a) used Richards Equation and a diffusion-convection equation to model water and Phosphorus flow through soil in order to measure absorption into the surrounding plant root system. Since variations in the horizontal change for root length density are marginal at the field scale compared to the vertical change, the model assumed that the soil is homogeneous and ignored movement of water and Phosphorus on the horizontal. (Roose and Fowler, 2004a). The findings in their research showed that fertilizer movement through soil is very slow and move at an approximate speed of 0.004cm per day thus the effect is felt after a longer time period. In addition to the results, the rate of fertilizer uptake by plant roots is dependent on the amount of rainfall and its absorption in the shoot is faster than in the roots.

Soil resistance, plant root distribution, climate change demand, and often a compensation term are all sink terms in the Richards equation (Javaux, et.al., 2013), and all of these factors influence fertilizer movement through soil. The solute transport model uses a mixed Eulerian–Lagrangian approach to the solution of the advective–dispersive-reactive transport equation (Chiang and Kinzelbach 2000). Modflow is a completely distributed model that measures ground water flow based on aquifer properties. It uses finite-difference approximations to solve the three-dimensional groundwater flow equation. The aquifer must be divided into cells for the finite-difference process to work, and the aquifer properties must be considered to be uniform. In each cell, the unknown head is measured at a point or node in the cell's center (Conan, et.al.,2003)

Models that describe and explain the processes relating to water and plant growth in the soil are used to generate numerical results in this study. Soil-Water-Atmosphere-Plant (SWAP) (Feddes, et.al., 1978) is the first version of the model, and recent versions such as SWAP version 3.2.36 by Kroes, et.al., (2009) and SWAP version 4.0.1 by Kroes, et.al., (2009) have been released on the internet (Swarp.wur.nl) (Kroes, et.al., 2017a). World Food Studies (WoFoS) is the other model, which is focused on crop growth models for a future production situation. Recent versions of WOFOST are available on the internet (wofost.wur.nl), and De Wit, et.al., (2018a) provide an overview of 25 years of WoFoS modeling. In advanced soil hydrological models, the use of more complex Richards and Convection-dispersion equations for unsaturated flow of water and solutes is popular (Vereecken, et.al., 2016).

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 Introduction

Water flow is linked to the transportation of different chemical substances in soil. Water dissolves quite a number of the compounds in soil environment, and they are transferred through the soil. Some compounds, on the other hand, remain insoluble in water and must be transported alongside water. Substances may not interact in any way with the soil system that surrounds it and make no changes over time, or either they interact with the soil system and change as a result of reactions involving chemicals, transformations in microbiology, and other factors. Solute transport is either conservative (the mass of the transported solute stays constant) or nonconservative because the mass of the transported solute changes and decreases because of adsorption, decomposition, nitrification, and valorization.

In this analysis, we developed a one-dimensional model perpendicular to the surface that can describe water-fertilizer movement, which is used to solve the Richards equation implicitly for unsaturated porous media using the finite element method for volume control, yielding water-fertilizer transport and general expressions for initial and boundary conditions. The rule of conservation of matter governs all water-solute movement in porous media (soil). The general governing flow equation for incompressible water transport in porous media is Richards' equation, which is obtained by combining Darcy's law and the continuity equation. The Richards Equation is given below according to Roose et.al.,(2004); Priesack et.al.,(2009); Simunek et.al.,(2008):

$$\frac{\partial\theta}{\partial t} = \nabla.\left[K(h)\nabla(h)\right] - \frac{\partial K(h)}{\partial z} - S(h) \quad (3.1)$$

A sink term is incorporated in the flow equation in order to account for water absorption by plant roots as a result of water and the effects of salinity. It is possible to accept both recognized and uncompensated water absorption by plant roots. Since the concentration of nutrients around the root surface determines plant growth rate, it can be presumed that water-fertilizer absorption by plant roots has a direct effect on plant growth. We therefore relate the Richards flow equation to the one-dimensional equation of contaminant transport through soil in groundwater then link it to the model of plant growth through the sink term which represents root uptake of water-fertilizer from soil.

$$S(h) \propto f(R) = \frac{dL}{dt} \quad (3.2)$$

$$f(R) \propto C \quad (3.3)$$

3.2 Fundamental Groundwater Flow Equations

3.2.1 Darcy Law

Henry Darcy, who was the publisher of the findings of his experimental research work in 1856, is credited with establishing the theory of groundwater flow. He discovered that total discharge Q is directly proportional to area of cross-section A, inversely proportional to length Δs, and proportional to the head difference $\emptyset_1 - \emptyset_2$, mathematically written in the equation called Darcy's equation:

$$Q = -K\,A\,\frac{\emptyset_1 - \emptyset_2}{\Delta s} \quad (3.4)$$

Dividing both sides of the equation (3.4) by A, we obtain the quantity $\frac{Q}{A}$ which is called specific discharge q often referred to Darcy velocity.

If $\emptyset_1 - \emptyset_2 = \Delta\emptyset$ and $\Delta s \neq 0$, the equation is converted into:

$$q = -K\frac{d\emptyset}{ds} \quad (3.5)$$

According to this equation, the specific discharge q, is direct in proportion to the hydraulic gradient K. Darcy's velocity is another name for the particular discharge. The actual flow velocity (the rate of seepage), V is written mathematically as V= $\frac{Q}{n.A} = \frac{q}{n}$ where n denotes the soil porosity and V denotes the seepage velocity, which is invariably greater than q. In reality, the flow almost never has just one dimension. Darcy's law in its broadest sense is applied, with the assumption that the hydraulic conductivity K is constant in all directions:

$$q_x = -K\frac{\partial\emptyset}{\partial x}, \quad q_y = -K\frac{\partial\emptyset}{\partial y}, \quad q_z = -K\frac{\partial\emptyset}{\partial z} \quad (3.6)$$

These equations are written as follows for anisotropic materials:

$$q_x = -K_{xx}\frac{\partial\emptyset}{\partial x} - K_{xy}\frac{\partial\emptyset}{\partial y} - K_{xz}\frac{\partial\emptyset}{\partial z} \quad (3.7a)$$

$$q_y = -K_{yx}\frac{\partial\emptyset}{\partial x} - K_{yy}\frac{\partial\emptyset}{\partial y} - K_{yz}\frac{\partial\emptyset}{\partial z} \quad (3.7b)$$

$$q_z = -K_{zx}\frac{\partial\emptyset}{\partial x} - K_{zy}\frac{\partial\emptyset}{\partial y} - K_{zz}\frac{\partial\emptyset}{\partial z} \quad (3.7c)$$

3.2.2 Continuity Equation

Darcy's law offers three motion equations for the given four unknowns $(q_x, q_y, q_z, \emptyset)$. The basic physical theory of mass conservation must be fulfilled by the flow phenomenon.

When a simple a soil component (block of soil) is filled with water or some other fluid, regardless of the flow pattern, no mass can be gathered or lost. According to the conservation principle, the total sum of the three measured quantities (mass flow) must equal zero.

$$\frac{\partial(\rho q_x)}{\partial x} + \frac{\partial(\rho q_y)}{\partial y} + \frac{\partial(\rho q_z)}{\partial z} = 0 \quad (3.8)$$

When the density is constant because the fluid is incompressible, the equation reduces to:

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} = 0 \quad (3.9)$$

The equation (3.9) above is presented in cartesian coordinates, which is known as the Equation of Continuity.

Darcy's law is substituted into the equation of continuity according to Richards (1931), yielding:

$$\frac{\partial^2 \emptyset}{\partial x^2} + \frac{\partial^2 \emptyset}{\partial y^2} + \frac{\partial^2 \emptyset}{\partial z^2} = 0 \text{ or } \nabla^2 \emptyset = 0 \quad (3.10)$$

3.3 Transport of Fertilizer contaminants through Soil in Groundwater

The most common source of pollution to groundwater under agricultural lands is fertilizers. As a fertilizer is injected into groundwater, it spreads out and travels with it due to advection (caused by groundwater flow), dispersion (caused by mechanical mixing), and molecular diffusion. The mathematical relationships that exist between these processes as presented by Javandel I et.al., (1984) are as follows:

$$\frac{\partial}{\partial x_i}\left[D_{ij}\frac{\partial C}{\partial x_j}\right] - \frac{\partial}{\partial x_i}(C.V_i) - \frac{C'W'}{n} = T\frac{\partial C}{\partial t} \quad (3.11)$$

Where;

$$V_i = -\frac{K_{ij}}{n} \cdot \frac{\partial h}{\partial x_j}$$

$$T = \left[1 + \frac{\rho_b K_d}{n}\right]$$

C represents concentration of contaminant, V_i represents Normal pore water velocity (seepage) in the direction x_i, D_{ij} denotes the dispersion coefficient, K_{ij} denotes the hydraulic conductivity, C' is the fertilizer concentration in the sink fluid, W' denotes flow rate in volume as a percentage of the sink, n represents the amount of porosity in impact, h is the hydraulic head, T denotes the factor of retardation and x_i denotes the cartesian co-ordinate. The equation that describes fertilizer transport in groundwater is represented in one dimensional time-dependent equation:

$$D_L \frac{\partial^2 C}{\partial x^2} - V\frac{\partial C}{\partial x} = T\frac{\partial C}{\partial t} \qquad (3.12)$$

The above equation (3.12) can be rearranged and written as:

$$\frac{\partial C}{\partial t} + \frac{V}{T}\frac{\partial C}{\partial x} = \frac{D_L}{T}\frac{\partial^2 C}{\partial x^2} \qquad (3.13)$$

Substituting $u = \frac{V}{T}$ and $d = \frac{D_L}{T}$ in equation (3.13) above, we get:

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = d\frac{\partial^2 C}{\partial x^2}, \; 0 < x < L(t), t \geq 0 \qquad (3.14)$$

The following initial and boundary conditions as published by Alberts et.al., (2002) apply:

Initial condition C(x,0) = 1 and boundary conditions; C(x = 0, t) = 1 ,   C(L) = C(x = L, t) such that d$\frac{\partial C(L)}{\partial x} = -g(R)C(L)$ .... (3.15)

h$\frac{dR}{dt} = g(R)C - \sigma R, R(0) = 0$ ....(3.16)

We now consider piecewise constant Function, f (R) and a piecewise linear function, g (R)

Figure 3.1: shows a plot of g (R) function against R

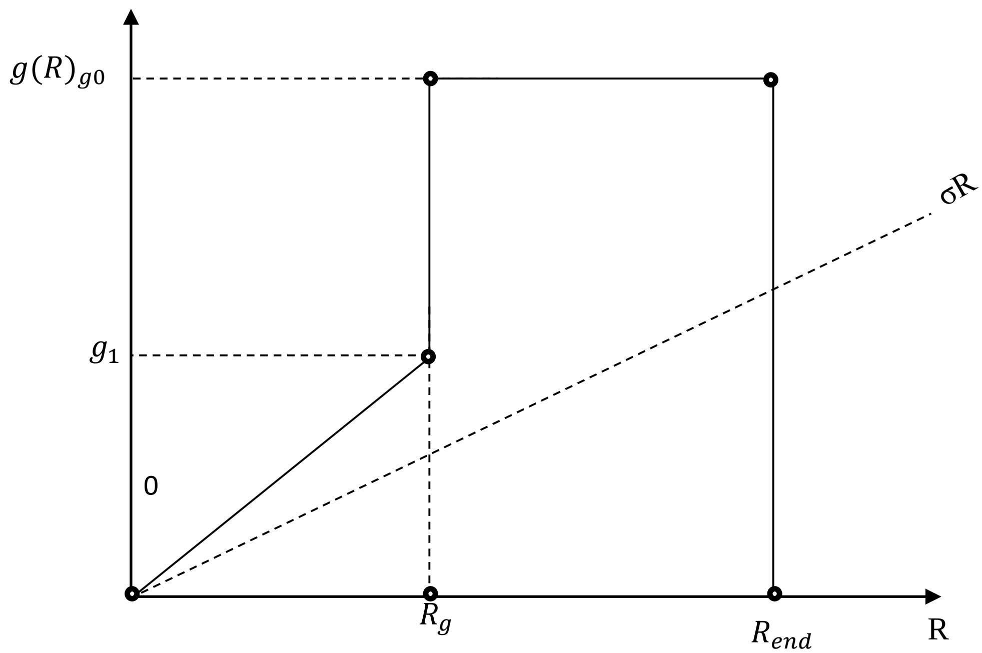


Figure 3.2: A plot of f (R) function against R

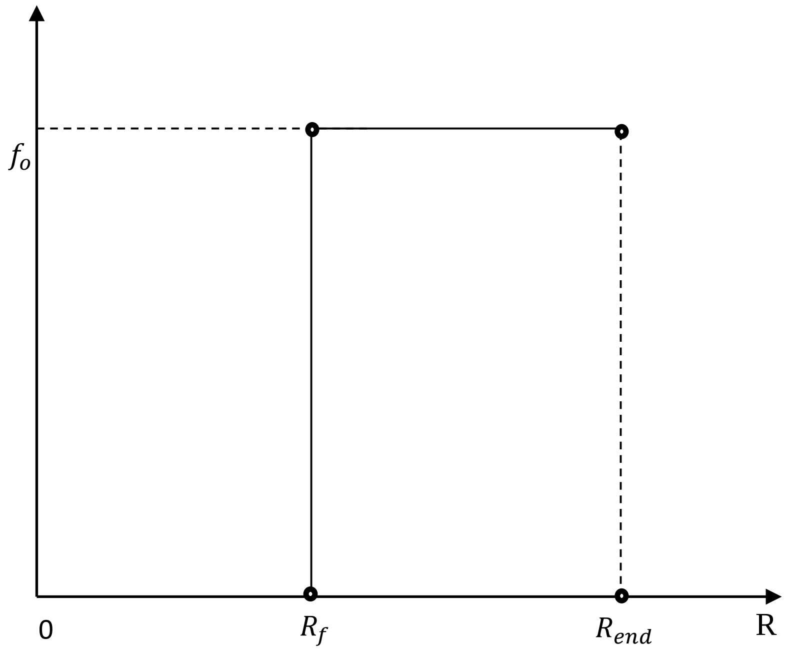
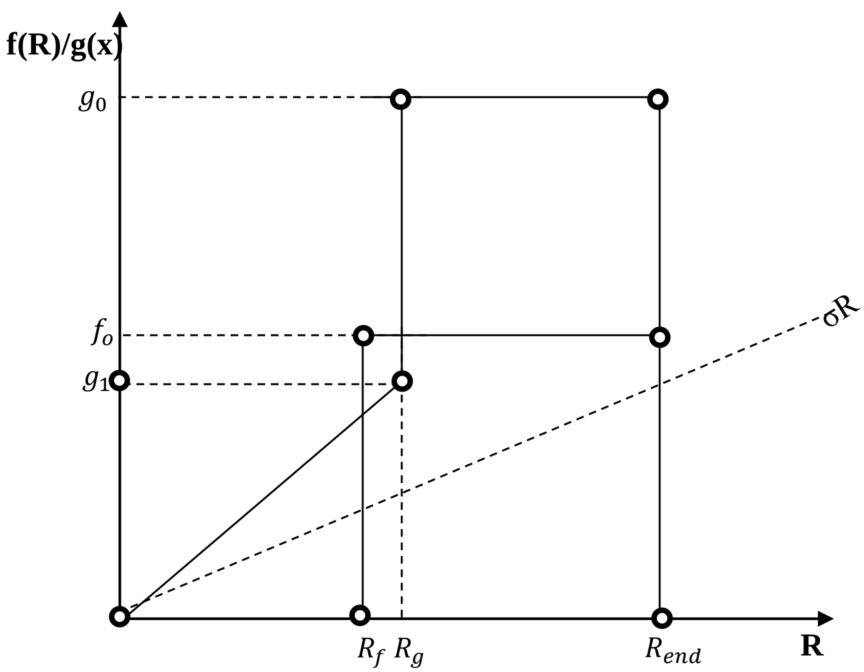


Figure 3.3: Graph of g (R) and f (R) combined functions against R



3.4 Analytical Descriptions

From equation (3.2) using Figure3.2;

$$\frac{dL}{dt} = f(R) = \begin{cases} 0, & 0 < R < R_f \\ f_{0,} & R_f < R < R_{end} \end{cases} \quad for\ 0 < t < t_f \text{..........} (3.17)$$

From equation (3.15) using Figure3.1;

$$\frac{dC}{dt} = -u\frac{\partial C}{\partial x} + d\frac{\partial^2 C}{\partial x^2}, 0 < x < L(t), 0 < t < t_f \text{..........} (3.18$$

Subject to;

Initial condition; C(x,0) = 1 and

Boundary conditions; C(x = 0, t) = 1, C(L) = C(x = L, t) = 1 such that

$$\frac{\partial C(L)}{\partial x} = -\frac{g(R)\,C(L)}{d} = \begin{cases} -\frac{g_1(R)\,C(L)}{R_g d} & ,0 < R < R_g \\ g_0 & ,R_g < R < R_{end} \end{cases} \text{..........} (3.19)$$

Similarly, from equation (3.16), we have

$$\frac{dR}{dt} = \frac{g\,(R)C}{h} - \frac{\sigma R}{.h} = \begin{cases} \frac{g_1(R)\,C}{R_g h} & ,0 < R < R_g \\ g_0 & ,R_g < R < R_{end} \end{cases} \text{..........} (3.20)$$

3.5 Discretization of the differential equations

3.5.1 Explicit Finite Difference Method

Using forward difference at time t_n and a second order central difference for the space derivative at position x; (Forward Time Centered Space) applied to diffusion problem e.g

$$\frac{\partial u}{\partial t} = \propto \frac{\partial^2 u}{\partial x^2}$$

The FTCS scheme is given by:

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = \frac{\propto}{\Delta x^2}\,(u_{i+1}^n - 2u_i^n + u_{i-1}^n)$$

Letting r= $\frac{\propto \Delta t}{\Delta x^2}$ we get: $u_i^{n+1} = u_i^n + r\,(u_{i+1}^n - 2u_i^n - u_{i-1}^n)$

Which is stable where r = $\frac{\propto \Delta t}{\Delta x^2} \leq \frac{1}{2}$

From equation (3.18) we use Explicit Finite Difference Method to solve as follows:

When 0< $R < R_f$

L (k + 1) = L(k) + Δt$R^0(0)$..........(3.21a)

When $R_f < R < R_{end}$

L (k + 1) = L(k) + Δt$R^0(f_0)$..........(3.21b)

From equation (3.19);

$$\frac{C(i,k+1) - C(i,k)}{\Delta t} = -u\,\frac{[C(i+1,k) - C(i-1,k)]}{2\Delta x} + \text{d}\frac{[C(i+1,k) - 2C(i,k) + C(i-1,k)]}{\Delta x^2}$$

$C(i,k+1) = -r_1[C(i+1,k) - C(i-1,k)] + r_2\,[C(i+1,k) - 2C(i,k) + C(i-1,k)] + C(i,k)$..........(3.22)

Where $r_1 = \frac{u\Delta t}{2\Delta x}$ and $r_2 = \frac{d\Delta t}{(\Delta x)^2}$

Rearranging equation (3.22) by collecting like terms we obtain the equation below;

$C(i,k+1) = (r_1 - r_2)\,C(i+1,k) + (1 - 2r_2)\,C(i,k) + (r_2 + r_1)\,C(i-1,k)$..(3.23)

Which can be simplified as follows;

$C(i,k+1) = \propto_1 C(i+1,k) + \beta C(i,k) + \propto_2 \; C(i-1,k)$..(3.24)

For $i = 2, \ldots\ldots\ldots, M+1$ $\quad k = 1; \; N+1$

Where $\propto_1 = r_1 - r_2, \beta = 1 - 2r_2$ and $\propto_2 = r_2 + r_1$

When $0 < R < R_g$, equation (3.24) is subjected to;

$C(i,1) = 1$ for $1 < i < M+1$

$C(i,k) = 1$ for $1 < k < N+1$

But from equation (3.19), first part;

$\frac{C(i+1,k) - C(i-1,k)}{2\Delta x} = \frac{-g_1 \, C(i,k) \, R(k)}{R_g d}$ when $i = M+1$;

At right boundary;

$$C(M+2,k) = \frac{-2\Delta x g_1 C(M+1,k) R(k)}{R_g d} + C(M,k)$$

From equation (3.24), we have;

$C(M+1, k+1) = \propto_1 \left[\frac{-2\Delta x g_1 C(M+1,k) \, R(k)}{R_g d} + C(M,k)\right] + \beta C(M+1,k) + \propto_2 C(M,k)$

....(3.25)

When $R_g < R < R_{end}$

$C(i,1) = 1$ for $1 < i < M+1$ and

$C(1,k) = 1$ for $1 < k < N+1$

But from equation (3.19) second part;

When i = M+1,

$C\ (M+2, k) = 2\Delta x g_0 + C(M,k)$

From equation (3.24);

$C(M+1,k+1) = \propto_1 [2\Delta x g_0 + C(M,k)] + \beta C(M+1,k) + \propto_2 C(M,k)$.........(3.26)

From equation (3.19);

When $0 < R < R_g$, $\frac{R(k+1) - R(k)}{\Delta t} = \frac{g_1 R(k) C(i,k)}{R_g h} - \frac{\sigma R(k)}{h}$

Multiplying through by Δt and adding R(k) to both sides of the equation, we obtain;

$R(k+1) = \frac{\Delta t g_1 R(k) C(i,k)}{R_g h} - \frac{\Delta t \sigma R(k)}{h} + R(k)$...(3.27)

When $R_g < R < R_{end}$;

R(k+1) = R(k) + $\Delta t g_0 (R^0)$ ....(3.28)

We now program equations (3.21a), (3.21b), (3.24), (3.26) and (3.27) in MATLAB, where the codes are presented in the appendix.

CHAPTER FOUR

RESULTS AND DISCUSSIONS

4.1 Introduction

Stephens, W. (2002) discussed various crop growth models and the developments of these models for use in numerical integration and computer programming. Most computer programs describing growth dynamics relate environmental factors to variables operating on spatial and temporal functions. We have used the following arbitrary values for the parameters to come up with tables and programmed in MATLAB to obtain the graphs.

Parameters

u=0.01; velocity profile

d=0.001; diffusion coefficient

Rg=0.01; critical value for g(R)

Rf=0.009; critical value for f(R)

g1=0.0001;

g0=0.001;

f0=0.01; positive constant

h=0.001; soil-water pressure head

x0=0; initial distance

M=5; M is number of subdivisions along x(distance) axis

N=10; N for number of subintervals along t(time) axis

Q=10; Q is number of subintervals along R(GM-factor) axis

L0=0.1; initial length

Lf=1; maximum length for which concentration is to be calculated

dx = (Lf-L0)/M ; distance interval

x = [0:M]'*dx; values of distance (x)

t0=0; initial time

T=2 ; maximum time (final time)

dt = (T-t0)/N; time interval

t = [0:N]*dt; values of time (t)

R_0=0; Rend=0.05; initial and final GM-factor

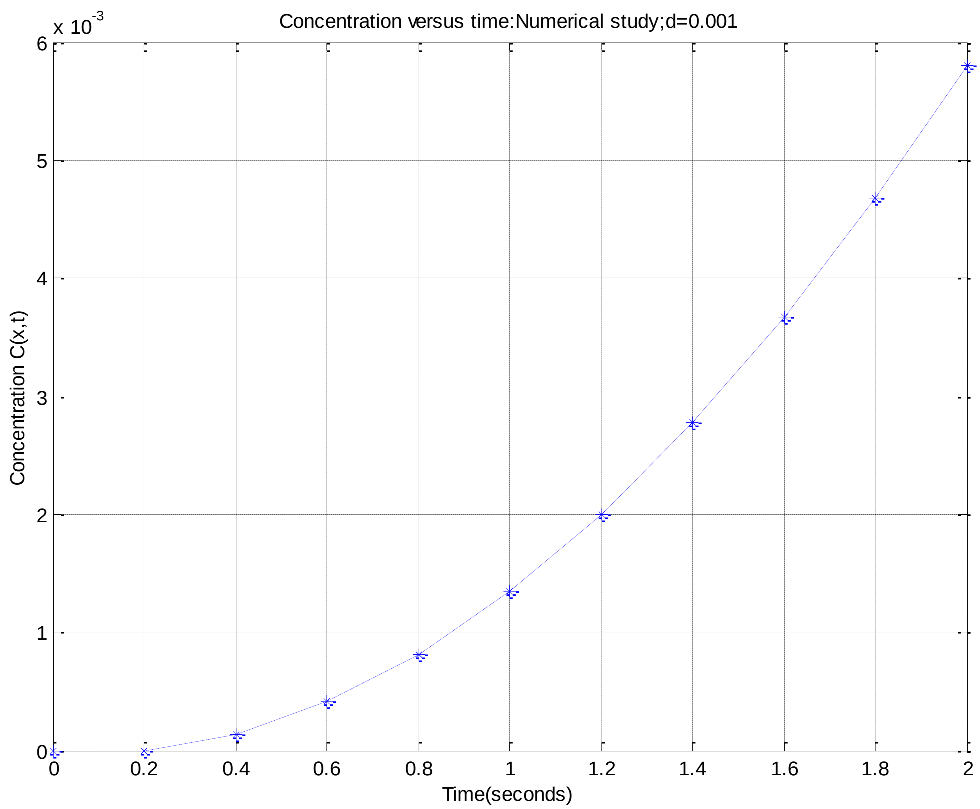


Figure 4.1 Graph of concentration of fertilizer against time when d=0.001

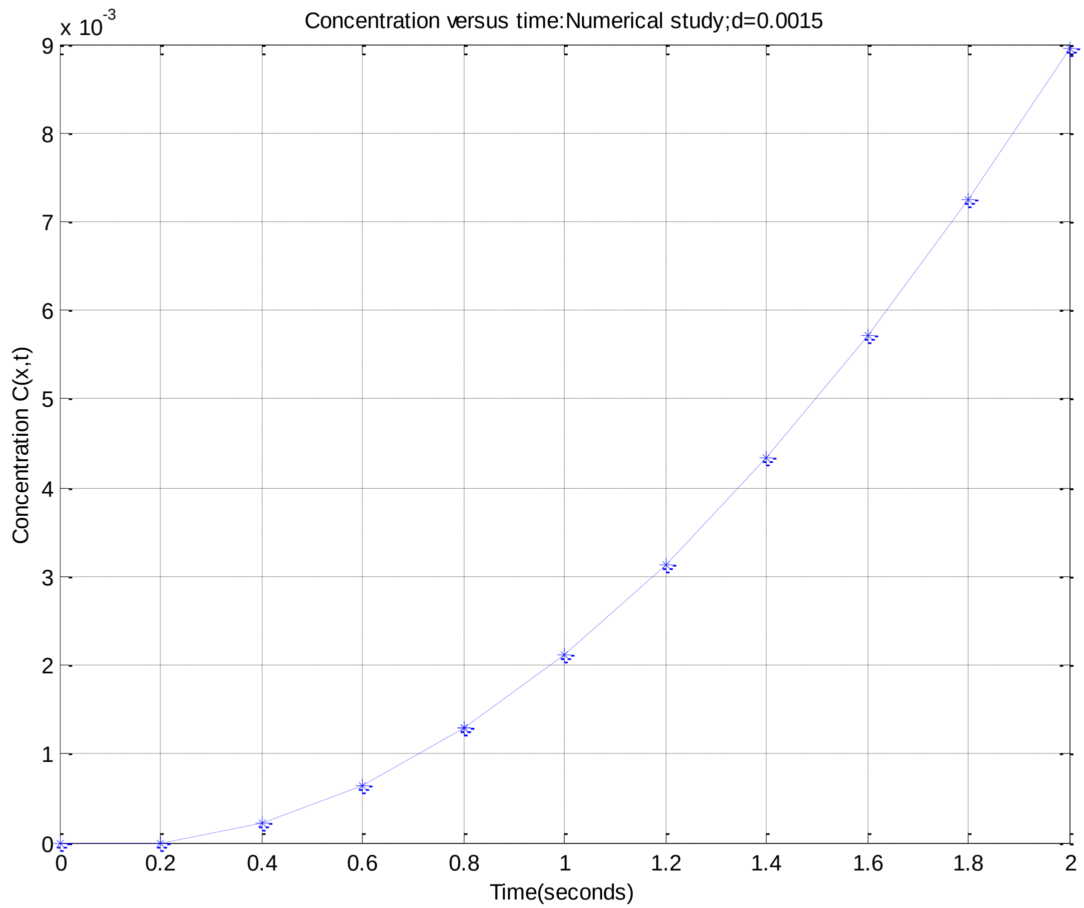


Figure 4.2 Graph of concentration of fertilizer against time when d=0.0015

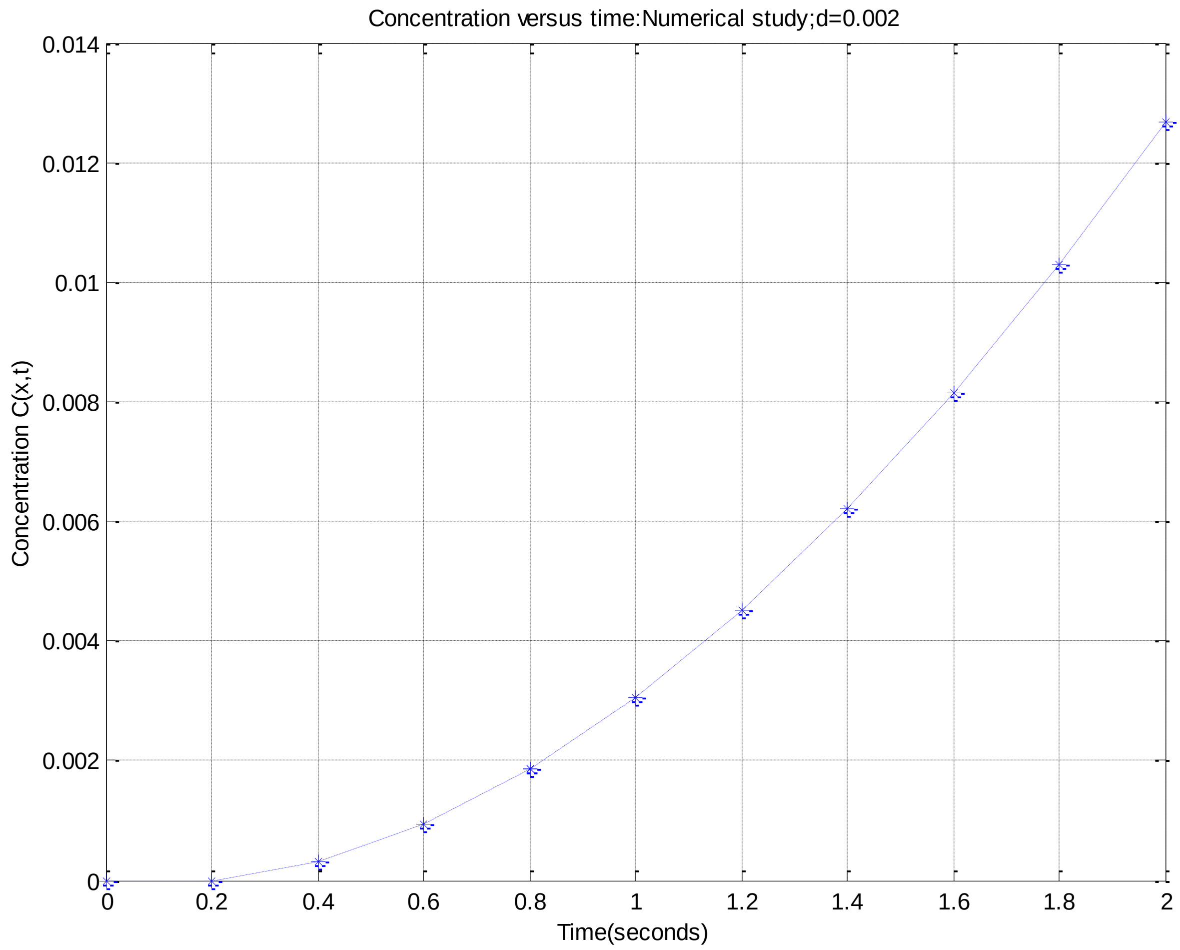


Figure 4.3 Graph of concentration of fertilizer against time when d=0.002

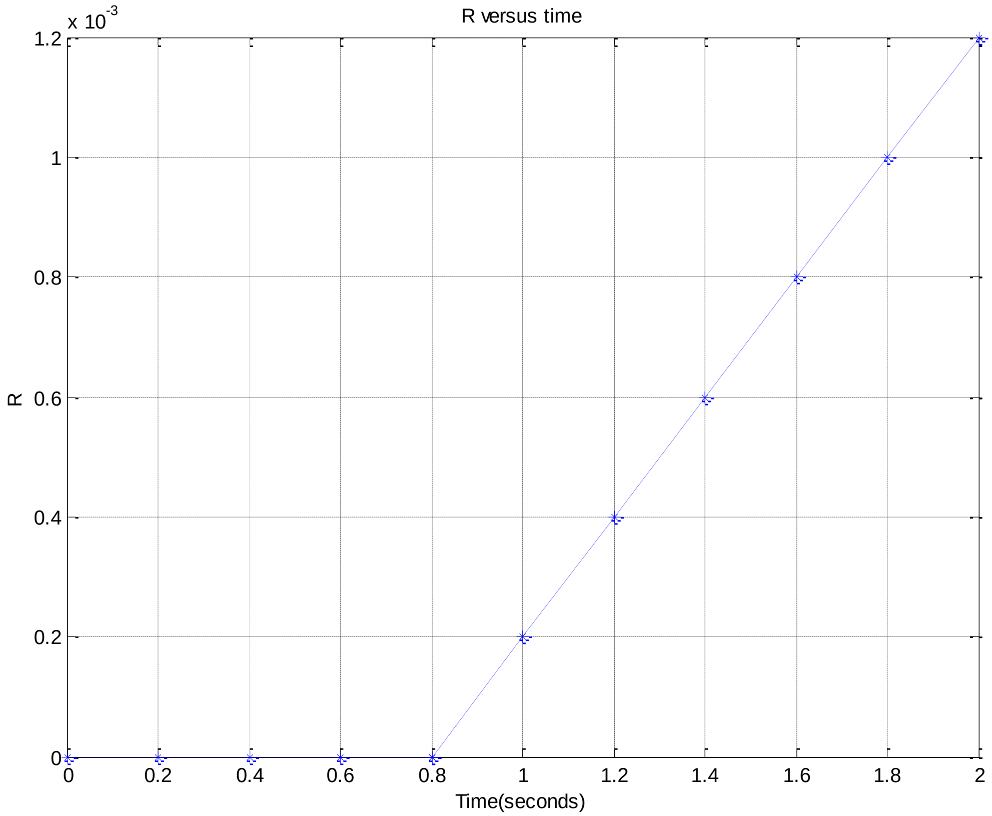


Figure 4.4 Graph of concentration of growth factor, R over a given time interval.

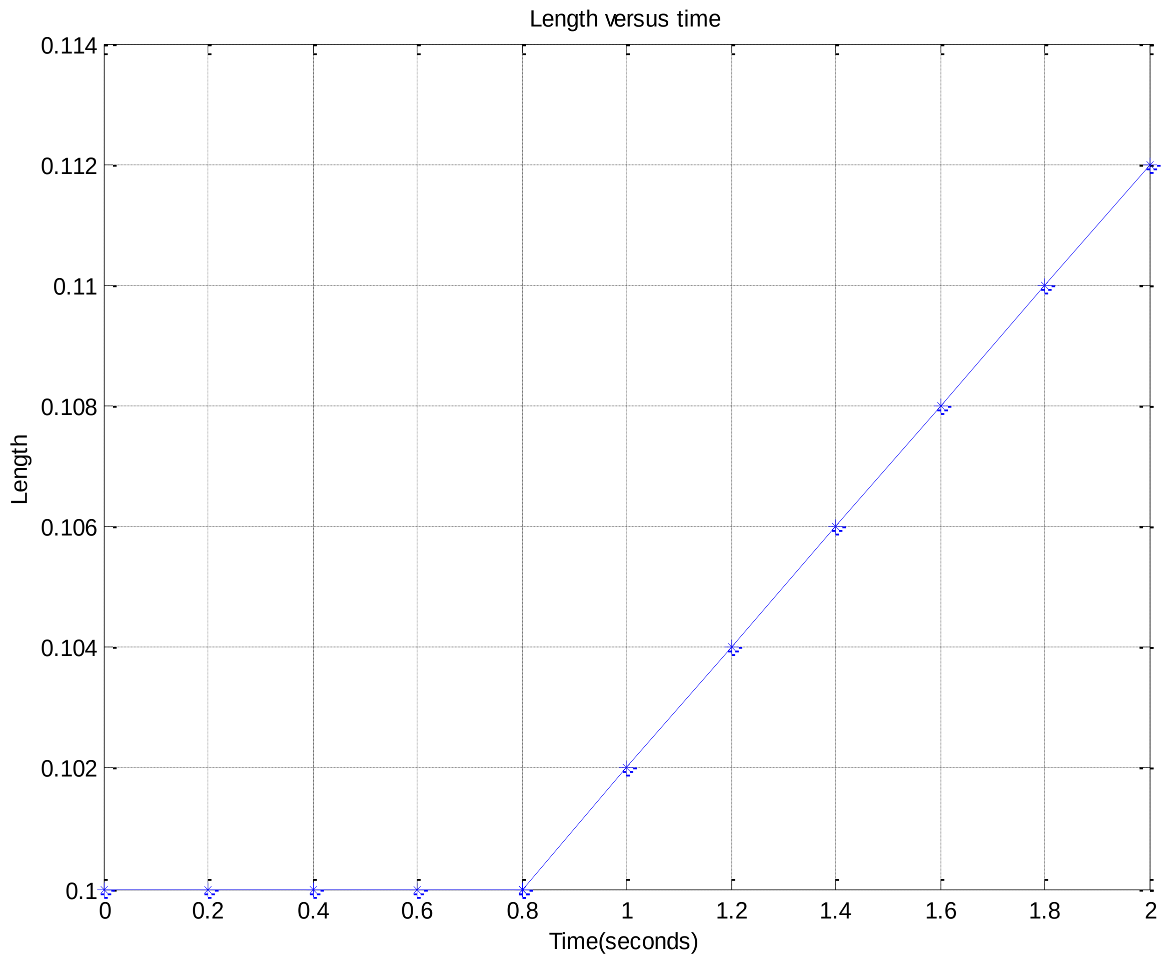


Figure 4.5 Graph of Length, L of growth of plant over a given time period.

Discussion

In figures 4.1-4.3, the results from the simulation values show the relationship between concentration of fertilizer absorbed by plant roots with time of simulation. From the graph, the concentration is zero from time 0 to time 0.2 before rising gradually. This is because the concentration is dependent on the flow through soil and accumulation on the root surface of the plant that takes sometime before it is realized. The results in the three figures also indicate that an increase in diffusion coefficient causes an increase in concentration of fertilizer in soil over a given time period. This is clearly seen on comparing the concentrations from the three graphs at time t=2.

The Fig 4.4 shows how the amount of growth mitosis factor; R varies with time which is linked to the concentration of fertilizer in the plant as it depends on the concentration. The amount of R remains at zero first before beginning to rise just as the concentration of fertilizer and rises also when the concentration of fertilizer rises. This is explained by the relationship of the two factors in the equations of flow and plant growth.

The last figure (fig 4.5) is a graph showing how length of plant apex varies over a given time interval. Length of growth does not change between t=0 and t=0.8 until the amount of growth mitosis factor; R reaches a critical value where a linear relationship is realized. When the amount of R increases, the length of growth also increases with respect to time.

From the results collected and analyzed above, we find that concentration of fertilizer solution, C in soil increases gradually with time, t. This in turn causes an increase in growth mitosis factor, R and hence increase in the vertical length of the plant. This shows that fertilizer concentration in soil accumulates over a period of time and this causes the plant to acquire metabolites responsible for growth. As a result, the plant grows on the apex by increasing in length.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

Food shortage in parts of the Kenya has been a problem for years due to environmental factors and poor innovation. With good agricultural practices that apply scientific research to ensure crops thrive well, the problem can be solved. One of the ways is through Mathematical modelling to simulate and explain agricultural processes by formulating and solving equations of fluid flow and crop growth. In this research, equations of solute(fertilizer) transport through soil in underground water was studied and its findings used to model plant growth. The solute concentration taken as fertilizer concentration in soil media over a given time increases causing an increase in growth mitosis factor and finally results in increase in apical(vertical) length of the plant. Such scientific findings play a big role in explaining processes that can improve agricultural practices especially the use of both organic and inorganic fertilizers.

5.2 Recommendations

From our research, concentration of fertilizer and the growth mitosis factor coupled with length of growth of plant were emphasized leaving other factors out. In future research, the velocity of flow of fertilizer solution through soil medium caused by amount of rainfall can be modelled and studied to see the effect of the speed of flow of the mixture on plant growth. Furthermore, research can be carried out to know the effect of dry climate on the flow of nutrients through soil. Again, further research can be carried out to check how a specific fertilizer flows through a given type of soil and its effect plant/crop growth.

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APPENDICES

Appendix I: TABLES OF VALUES

The values given in the tables below are not experimental values but arbitrary values used for simulation.

Table 4.1: Effect of concentration of fertilizer solution due to flow over a given time interval and diffusion coefficient; d=0.01

	x values					
Time	0	0.18	0.36	0.54	0.72	0.9
0	1	0	0	0	0	0
0.2	1	0.0117	0	0	0	0.9877
0.4	1	0.0233	0.0001	0	0.0006	0.9877
0.6	1	0.0348	0.0004	0	0.0012	0.9877
0.8	1	0.0461	0.0008	0	0.0018	0.9877
1	1	0.0572	0.0013	0	0.0024	0.9877
1.2	1	0.0682	0.002	0	0.003	0.9877
1.4	1	0.0791	0.0028	0.0001	0.0035	0.9877
1.6	1	0.0899	0.0037	0.0001	0.0041	0.9877
1.8	1	0.1005	0.0047	0.0001	0.0047	0.9877
2	1	0.111	0.0058	0.0002	0.0052	0.9877

Table 4.2: Effect of concentration of fertilizer solution due to flow over a given time interval and diffusion coefficient; d=0.0015

	x values					
Time	0	0.18	0.36	0.54	0.72	0.9
0	1.0000 0	.0000 0	.0000 0	.0000 0	.0000 0	0
0.2	1	0.0148	0	0	0	0.9815
0.4	1	0.0294	0.0002	0	0.0036	0.9816
0.6	1	0.0436	0.0007	0	0.0072	0.9816
0.8	1	0.0576	0.0013	0.0001	0.0107	0.9816
1	1	0.0714	0.0021	0.0001	0.0141	0.9817
1.2	1	0.0849	0.0031	0.0002	0.0175	0.9818
1.4	1	0.0981	0.0043	0.0003	0.0208	0.9819
1.6	1	0.1112	0.0057	0.0004	0.0241	0.9819
1.8	1	0.1239	0.0073	0.0006	0.0273	0.982
2	1	0.1365	0.009	0.0008	0.0304	0.982

Table 4.3: Effect of concentration of fertilizer solution due to flow over a given time interval and diffusion coefficient; d=0.002

	x values					
Time	0	0.18	0.36	0.54	0.72	0.9
0	1.0000 0	.0000 0	.0000 0	.0000 0	.0000 0	0
0.2	1	0.0179	0	0	0	0.9753
0.4	1	0.0354	0.0003	0	0.0066	0.9755
0.6	1	0.0524	0.0009	0.0001	0.0131	0.9756
0.8	1	0.069	0.0019	0.0002	0.0194	0.9757
1	1	0.0852	0.0031	0.0003	0.0255	0.976
1.2	1	0.101	0.0045	0.0005	0.0315	0.9761
1.4	1	0.1165	0.0062	0.0008	0.0374	0.9763
1.6	1	0.1315	0.0081	0.0012	0.0431	0.9764
1.8	1	0.1462	0.0103	0.0016	0.0487	0.9765
2	1	0.1606	0.0127	0.002	0.0542	0.9767

Appendix II: MATLAB CODE

```
% Numerical Study of effect of fertilizer concentration on plant growth

%
###########################################################################

% solve C_t+ u*C_x=d*C_xx for 0 <= x <= xf, 0 <= t <= T

% Initial Condition: C(x,0) = =it0(x)

% Boundary Conditions: C(0,t) = g0(t)=bx0(t)  (left  BC)

%                      C(xf,t)= g1(t)= bxf(t) (right BC)

clc,clf,clear all,close all% clear screen,clear figure,clear all declared variables, close all figures

% Parameters

u=0.01;

sigma=0.009;

Rg=0.01;       % critical vaue  for g(R)

Rf=0.009;      % critical vaue  for f(R)

g1=0.0001;

g0=0.001;

f0=0.01;

h=0.001;       % soil-water pressure head

x0=0;          % initial distance

M=5;           % M = # of subintervals along x(distance) axis

N=10;          % N = # of subintervals along t(time) axis
```

```
Q=10;        % Q = # of subintervals along R(GM-factor) axis

L0=0.1;      % initial length

Lf=1;        % maximum length  which concertation is to be calculated

dx =      (Lf-L0)/M ; % distance interval

x = [0:M]'*dx;   % values of distance (x)

t0=0;        % initial time

T=2;      % maximum time (final time)

dt = (T-t0)/N;   % time  interval

t = [0:N]*dt;    % values of time (t)

R0=0;R_end=0.05;     % initial and final GM-factor

dR=(R_end-R0)/Q;

% First equation

% when 0<R<Rf

L(1)=L0;

L(N/2)=Lf/2;

   for k=1:N/2

      L(k+1)=L(k)+dt*0 ; % upper equation

   end

figure (1)

plot(t(1:N/2),L(1:N/2),'*-');                  % plot concentration against time

xlabel('Time(seconds)')                        % label x-axis
```

```
ylabel('Length')                          % label y-axis

title('Length versus time')                  % title of 2D graph

grid                                     % insert grid lines to graph

hold on

% when Rf<R<R_end

L(N/2)=L(k+1);

L(N+1)=Lf;

for k=(N/2):N+1

   L(k+1)=L(k)+dt*f0;  % lower equation

end

plot(t(N/2:N+1),L(N/2:N+1),'*-')                  % plot concentration against time

% Second equation

% when 0<R<Rg

C=ones(M+1,N+1);

R(1)=R0;

R(N/2)=Rg;

 for i = 1:M + 1

    for k=1:N/2

      R(k+1)=dt*g1*C(i,k).*R(k)/(Rg*h)-dt*sigma*R(k)/h+R(k);% upper equation

    end

 end
```

```
figure (2)
plot(t(1:N/2),R(1:N/2),'*:')          % plot concentration against time
xlabel('Time(seconds)')               % label x-axis
ylabel('R')                           % label y-axis
title('R versus time')                % title of 2D graph
grid                                  % insert grid lines to graph
hold on
 % when Rg<R<R_end
R(1)=Rg;
R(N+1)=R_end;
for k = (N/2):(N + 1)
   R(k+1)=dt*g0+R(k);   % lower equation
end
plot(t(N/2:N+1),R(N/2:N+1),'*:')          % plot R against time
for d=[0.001 0.0015 0.002]
r1=u*dt/(2*dx);r2=d*dt/(dx^2);alpha1=r2-r1;alpha2=r2+r1;beta=1-2*r2;
% Third equation
% when 0<R<Rg
R(1)=R0;
R(N/2)=Rg;
for i=1:M+1,C(i,1)=1e-44;end  %IC
```

```
for k=1:N+1,C(1,k)=1; end     %BC(left)

for k=1:N+1

   C(M+1,k)=alpha1*(-
2*dx*g1*C(M+1,k)*R(k)/(Rg*d)+C(M,k))+beta*C(M+1,k)+alpha2*C(M,k);

end %BC(rigth)

for k = 1:N                              % start of time loop

for i = 2:M                              % start of distance loop

   C(i,k+1)=alpha1*C(i+1,k)+beta*C(i,k)+alpha2*C(i-1,k);% Explicit Numerical scheme
used; see attached file

end                                      % end of distance loop

end                                      % end of time loop

figure (3)

plot(t(1:N/2),C(M/2,1:N/2),'*:')              % plot concentration against time

hold on

xlabel('Time(seconds)')                     % label x-axis

ylabel('Concentration C(x,t)')                 % label y-axis

title('Concentration versus time:Numerical study;') % title of 2D graph

grid                                     % insert grid lines to graph

hold on

%legend('d=0.001', 'd=0.0015','d=0.002')

% when Rg<R<R_end

R(1)=Rg;
```

```
R(N+1)=R_end;
for i=1:M+1,C(i,1)=1e-44;end  %IC
for k=1:N+1,C(1,k)=1; end   %BC(left)
for k=1:N+1
   C(M+1,k)=alpha1*(2*dx*g0+C(M,k))+beta*C(M+1,k)+alpha2*C(M,k);
end %BC(rigth)
for k = 1:N                                   % start of time loop
for i = 2:M                                   % start of distance loop
   C(i,k+1)=alpha1*C(i+1,k)+beta*C(i,k)+alpha2*C(i-1,k);% Explicit Numerical scheme used; see attached file
end                                    % end of distance loop
end                                    % end of time loop
figure (3)
hold on
plot(t(N/2:N+1),C(M/2,N/2:N+1),'*:')             % plot concentration against time
%legend('d=0.001', 'd=0.0015','d=0.002')
end
```

Appendix III: Similarity Report

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