MATHEMATICAL MODELLING ON THE IMPACT OF HOSPITALIZATION IN THE MANAGEMENT OF TYPHOID FEVER

BY

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DECLARATION

Declaration by the Student

This thesis is my original work and has not been submitted for any academic award in any institution; and shall not be reproduced in part or full, or in any format without prior written permission from the author and/or University of Eldoret.

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DEDICATION

This thesis is dedicated to my son, Bradley.

ABSTRACT

Typhoid fever disease is an infectious ailment which mostly leads to diarrhoea, headache, high fever and stomach pains. This disease is majorly caused by a bacterial infection known as Salmonella typhi. Typhoid fever has caused a significant burden in most developing countries hence a concern to the health sector. In this thesis, a mathematical model has been developed, and based on the ordinary differential equations; the mathematical model is analyzed quantitatively basing on the impact of hospitalization in the management of typhoid fever disease. Hospitals play a big role in the control of typhoid fever through their admission of patients and treatment; therefore, in this thesis a model is developed which explains the effect of increasing hospitalization. The invariant region is worked out in which the model solution is bounded so as to obtain the feasible solution of the set. The next generation matrix method is used to attain the basic reproduction number. The disease free equilibrium and the local stability of the disease free equilibrium determined. The numeric results obtained are determined graphically by use of maple simulation method. The results indicated that; the rate of hospitalization is inversely proportional to the rate of infections while there is a constant rise in the carrier population.

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CHAPTER ONE

INTRODUCTION

1.1 Background to the Study

The other name of typhoid fever is enteric fever. It is a potentially deadly ailment caused mostly by *Salmonella enterica* of serotype typhi and, to a lower extent, *Salmonella enterica* of serotypes Paratyphi A, Paratyphi B and Paratyphi C these terms are mainly used as a description to the main serotypes. (Brusch, J.L,2019). Typhoid fever symptoms include high fever, headache, stomach pain and either constipation or diarrhea. It incubates for a period of between 7 and 14 days. (WHO SAGE November 2007; and Muhammad A.K, 2017). It is commonly spread through contaminated foods or fluids such as water. Typhoid fever is endemic in most developing countries and is continuously becoming a public health problem and concern, despite recent improvement on water sanitation. (Lauria, D. T *et al.,* 2009). Typhoid fever disease causes not less than 600,000 fatalities every year in the world. The disease has always been underestimated and to some extent ignored, even though it is a serious health problem. (Benard Ivanoff *et al.,*1994) Due to high infectivity rate and increasing disease strain which is burdening, typhoid fever constitutes a major world health problem. However, the vaccine for typhoid fever remains the essential tool for proper management of the disease. Recently there have been two main types of vaccines. Where one of the vaccines is bases on the well defined subunit "virulence (vi) polysaccharide antigen" and the vaccine may be administered by either intramuscularly or subcutaneously whereas the other vaccine is administered by use of the live attenuated bacteria which is administered orally (Carlos A Guzman, *et al.,* 2006). Although many mathematical models have been developed, the models approached the typhoid fever under different aspects. However, these models did not take into account the effect of hospitalization in the management of typhoid fever disease in detail. From their studies, an SIR model was developed which contained hospitalization and home-based care compartments. The model emphasized hospitalization as a mode of reducing infections.

1.2 Problem Statement

Hospitalization is the process in which an individual suffering from typhoid fever is taken care of in a medical facility by qualified medical personnel. Hospital management of typhoid patients is not common in most areas; this is due to the stereotype that home based care is better compared to hospital management. Those taking care of typhoid patients at home end up endangering the patient's life as well as their lives through new infections of the typhoid fever. This has mainly lead to continous increment of the typhoid fever disease. An increase in typhoid fever tend to increase the infectiousness of the fever which poses a greater risk to the people living in congested areas. Congestion lead to increased contact rate of the infected and those who are disease free hence new infections arise.

In developing countries such as Kenya, specifically; people living in congested areas such as slums, they do not seek medical attention. As a result, it has become a killer disease. Most people in rural areas prefer herbal medication to conventional medicines, western medicine, bio-medicine etc, however, the herbalists may mis-diagnose the type of disease one is suffering from. Such people need proper sensitization on the importance of visiting hospitals for medication when they have typhoid symptoms. Hospitalization of typhoid patients may be hampered by; fewer hospitals in an area, may not be cost-effective and development of out-patient facilities. This thesis models hospitalization as a mode of reducing typhoid fever infections. The study guides on the importance of; increasing hospitals in an area and equiping them with enough beds, discouraging out-patient services for infectious diseases and subsidizing the cost of admitted patients. From the studies of different scholars, hospitalization was not discussed exhaustively hence in this model, an attempt has been made to incorporate hospitalization in the management of typhoid fever disease. The importance of hospitals and hospital management of patients in curbing or reducing the rates of infections and deaths due to the disease in the society, has been analyzed.

1.3 Objectives of the Study

1.3.1 General Objective

To develop and analyze a mathematical model that incorporates hospitalization in the control of typhoid fever outbreak.

1.3.2 Specific Objectives

1. Developing a mathematical model incorporating the impact of hospitalization and home based care in the treatment of typhoid fever.
2. To analyze the developed model by use of numerical simulation.
3. To determine the relationship between hospitalization and the infectiousness of typhoid fever over time.

1.4 Significance of the Study

The study is important in analyzing the relationship between hospitalization and the infectiousness of the typhoid fever, this analysis reduces the chance of a disease outbreak. Research has been done by many scholars on the modelling of typhoid fever, however, there is limited research on the modeling of typhoid fever considering the impact of hospitalization. There is prevalence of typhoid fever in areas with high population and are as well low income earners. In such areas poor hygiene, contaminated water or food is common. When an outbreak occurs, many people will get infected since the disease is highly infectious and can be fatal if control measures are not in place, this may lead to high burden to providers in families. The typhoid fever has led to high mortality rates in children under 5 years and also above 5 years in Kenya. The research model will assist public health officers to understand the dynamics of typhoid fever transmission therefore enabling them develop effective ways of handling patients. This research will reduce the prevalence of the disease and its infectiousness therefore the mortality rate will have reduced. The research will widen the potential of academic researches on the importance of incorporating hospitalization in the model as used. It further offers assistance to the governance in a country on the importance of having enough medical facilities in the case of typhoid fever outbreak.

CHAPTER TWO

LITERATURE REVIEW

Mathematical modelling of diseases is important in studying the trend of an infectious disease and making an analysis on how they can be curbed or reduced. An SIR model for spread of disease was formulated, the model consisted of Susceptible, Infective and Recovered which was used to determine the spread of an infectious disease over a given period of time (David and Lang; 2014). Most scholars have improved the SIR model for typhoid fever therefore providing normal and reasonable results.

Getachew Teshome Tilahun *et al.* (2017) proposed and analyzed a compartmental mathematical model which is non-linear and deterministic for the outbreak of typhoid fever which contained optimal strategies for control of typhoid with changing population. They developed a compartment model with five classes of $S - C - I - R - B_c$ where S-Susceptible, I Infected, C-Carrier, R- Recovered and B_c - Bacteria population, from these compartments they formulated equations from the mathematical model. From the equations developed, the invariant region was obtained within which the solution was bounded. From the same equations, they worked out the positivity of the solutions, the disease free equilibrium (stability both globally and locally), the endemic equilibrium and the basic reproduction number. They did the sensitivity analysis and made interpretations. In their design, they applied Pontryagin Maximum principle which contained prevention strategy via sanitation, the process of vaccination, that is, treatment by use of appropriate medicine with carriers being screened and proper hygiene considered. They discussed and concluded that treatment and prevention is a good cost-effective strategy to the disease eradication. They discussed a model of incorporating bacterial population in the control strategy, which was their main objective. From their discussion they worked out the basic reproduction number and found it less than one, implying that the disease free equilibrium becomes stable for both global and local. When the basic reproduction number is greater than one, it meant that the endemic equilibrium is stable locally and stable globally at equilibrium. They however did not consider hospital management of typhoid in their methodology. They obtained numerical results for analysis. However, from their research, hospitalization was not considered.

Peter O.J *et al.* (2017) on their research article of mathematical model used in control of typhoid fever, they used Lipchitz condition to test for uniqueness and existence of solution.

They further developed a compartment model with five classes P-S-I-T-R where P-vaccinated but loses protection over time, S-susceptible, I-infectious, T-treated and R-recovered. Their mathematical model was in the form of P-S-I-T-R from which they developed mathematical equations. Based on the developed equations, they determined the existence and the uniqueness of the solutions, they further worked out the states of equilibrium which determined their stability and did an estimate on the reproductive number. From the model equations they worked out the stability of the disease free equilibrium. Their discussions concluded that the disease can be controlled by ensuring the contact rate with infected people is minimized. They obtained basic reproduction number by use of next generation matrix and proved that disease free equilibrium is asymptomatically stable when the reproduction number is less than one, this implies that the disease will die out naturally. From their study, their main objective did not capture hospitalization.

Nthiiri J. K *et al.* (2016) on their mathematical model article, incorporated protection from infection of typhoid fever. The study formulated a model which based on the ordinary differential equation system which studied the dynamics of the typhoid fever and incorporated protection from infection. Nthiiri J. K *et al.* (2016) developed a compartmental model with four classes S-P-I-T where S is the Susceptible class, P is the Protected class, I am the Infected class and T is the Treated class. A mathematical model of P-S-I-T was formulated that came up with model equations. From these equations, they analyzed the model mathematically and worked out; the disease free equilibrium on stability, the endemic equilibrium and the basic reproductive number. They carried out the stability analysis and determined the condition which favoured the spread of typhoid fever disease. The findings on the numerical simulation showed low disease prevalence due to increase in protection. They made a conclusion that when typhoid fever is controlled effectively, it prevents fast progress to infection specifically in areas with scarce or less resources. In such areas, vaccination is deemed important. From their study, they majorly stressed on the incorporation of protection against infection of typhoid fever hence did not include hospital management of patients with the disease.

Muhammad A. K *et al.*, (2014) analysed mathematically a typhoid model which contained a saturated incidence rate. They developed a mathematical model of S- I- E -R where S is the susceptible sub-class, E is exposed sub-class, I is the infected sub-class and R is the recovered sub-class. Muhammad A. K *et al.* (2014) investigated local stability results and global stability. They assumed the population in the model mixed homogenously. From the model, they developed equations and tabulated parameters with the descriptions. From the equations formulated, they were able to work out the equilibrium in the absence of disease, basic reproduction number was worked out, the stability of the basic equilibrium was also worked out and the stability of the disease equilibrium calculated.

Numerical solutions were investigated for the proposed model since they had specified initial conditions. They obtained the solutions numerically by use of Runge-kutta method. They then presented the numerical results in the form of graphs.

They investigated local stability and global stability were concluded that the reproduction number is less than unity, the disease free equilibrium was both stable globally and locally at equilibrium. They however in their model did not take into account the impact of hospital management.

Peter O. J, *et al.*, 2018 in their model of typhoid fever by variation iteration method, sub-divided the population of humans to four compartments namely; the susceptible S(t), the infected carrier Ic(t), the infected I(t) and the recovered R(t). In their model, they did an assumption on direct typhoid fever transmission from the infected people to susceptible people. They incorporated the real biological phenomenon in which typhoid is mainly contracted from the bacteria in the environment through contaminated; foods, water and drinks, added a compartment W(t) which represented the environmental bacteria. Assumptions made were, susceptible individuals are infected with typhoid disease at the rate which is proportional to the population which is susceptible. The study employed variation iteration method to the non-linear system of differential equations which gave a description of their model in which they did an approximation of the solutions which was in a sequence of the time intervals. In order to illustrate its accuracy, the results obtained were compared with the classical fourth order of the Runge-kutta method.

Stephen Edward, Nkuba Nyerere 2016 developed a transmission model which was deterministic, the model captured vaccination, treatment and education campaigns control strategies. In their study, the human population N(t) is sub-divided into five compartments namely; the susceptible sub-class S(t), the infectious sub-class I(t), the vaccinated sub-class V(t), the carriers sub-class Ic(t) and the recovered sub-class R(t). the recruitment of individuals to the susceptible population is by either immigration or birth at a constant rate. They assumed that a certain proportion of susceptible individual's progress to carrier sub-class while the other remaining proportion of susceptible sub-class or individuals proceeded to the symptomatic infectious sub-class. The carriers may become symptomatic at a given rate or they can also die at a given rate due to typhoid fever disease. Infected individuals receive treatment and recover at a given rate. The recovered sub-class can also be susceptible once more meaning that after recovery, there is no permanent immunity. Individuals who are susceptible receive vaccination for protection against typhoid fever disease at a given rate. In their study, there was an education parameter which was catered in disadvantaging both carriers and the individuals who are symptomatic from spreading the typhoid fever disease. They calculated the disease free equilibrium and made a prove that it is locally asymptomatically stable when the reproduction number is less than one. They made a conclusion that to eradicate typhoid fever disease, education, vaccination and treatment are not the only modes. However, vaccines do not provide one with permanent immunity hence a possibility of the individual contracting the disease. The importance of hospitalization was not considered after a case of re-infection.

Peter O.J, *et al.* (2020) developed a mathematical model which comprised of four compartments namely the susceptible class, the carrier class and the recovered class. From these compartments, equations were formulated on the flow of the infectious disease, which were used for the determination of the existence and uniqueness of the model. They used the Lipchitz condition for the verification of the singularity of the solution. They further worked out the basic reproduction number and found it to be less than one which was an indication that every contagious person cannot cause an infection hence the disease always disappears. They performed numerical simulations and made a graphical description on the impact of long term through early treatment. They made a conclusion that through early treatment and detection, infectiousness of the typhoid fever disease may reduce. However, there was no specification on the kind of treatment, the factor of hospitalized care was not discussed.

From the authors discussed above, formulations were made which has guided in the study of the model. This model mainly gives reasonable and normal results. Assumptions are always made to improve the model analysis and its spread under different states or conditions. In most cases, the improved models tend to consist more efficiency compared to SIR model. In this model, an improvement from the SIR model was done to compare infectiousness of typhoid fever with hospitalization.

CHAPTER THREE

METHODOLOGY

3.1 Method of Formulation

The SIR model formulated was improved to include carriers, home-based care and hospitalized individuals having typhoid fever. The model developed gave reasonable and normal results. Assumptions are always made to improve the model analysis and its spread under different states or conditions. In most cases, the improved models tend to consist more efficiency compared to SIR model.

3.2 Model Description and Formulation

The deterministic mathematical model developed contained different compartments which capture the effectiveness of hospitalization and home-based care. The model developed contains six compartments from the human population (N); that is, susceptible (S), Infectious (I), Carriers (C), Home-based care (Hb), hospitalized (H) and Recovered (R) compartments.

The model developed is S-C-I-Hb-H-R, and the general form of the model is described by the diagram below in detail.

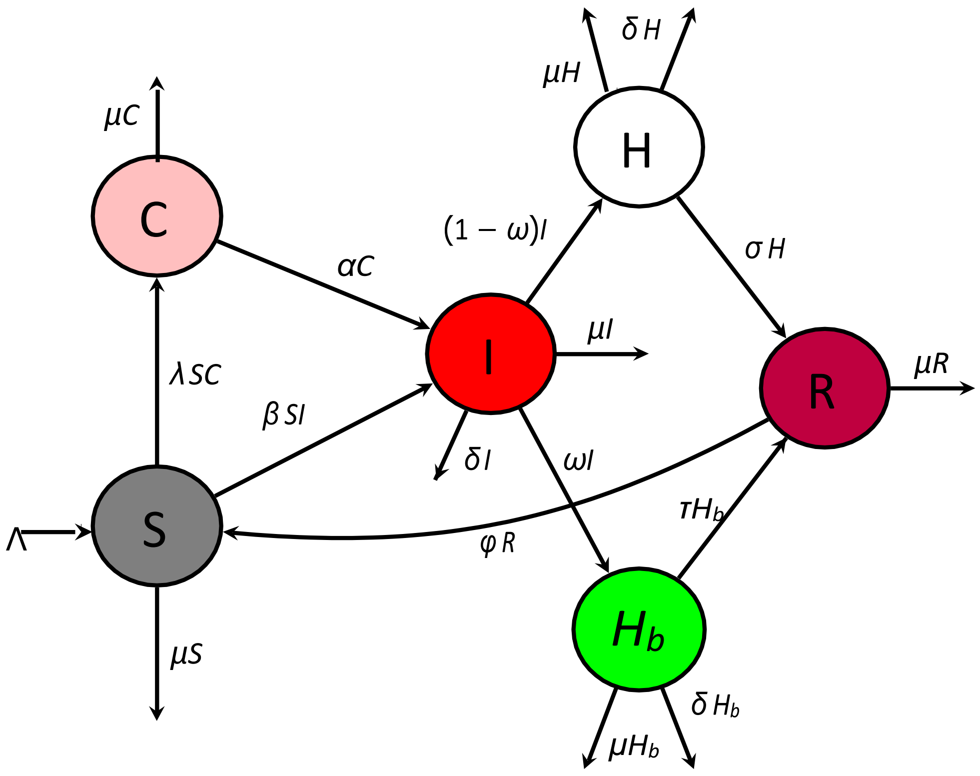

Figure 3.1 A compartmental model description including home-based care and hospitalization

Some arrows indicate the movement of individuals from one compartment to another while other arrows point outside the compartments. The arrows pointing outside the compartments indicate an exit from the population.

Susceptible individuals are those likely to be affected by the typhoid fever. The carriers are those individuals who are likely to transmit typhoid fever to others but do not suffer from the typhoid fever. The infectious individuals are those who have the disease and can easily transmit to other people. The home-based care individuals contracted the disease and are taking medication at home prescribed by qualified medical personnel, over the counter medication or from herbalists. The hospitalized individuals are those infected by the disease and admitted in a medical facility; attended by qualified health professionals. The recovered individuals are those who get well after a typhoid infection.

Susceptible individuals are recruited into the population at the rate of Λ, the recruitment of individuals is mainly by birth and to a lesser extent through immigration. The rate at which the susceptible become carriers is represented by λ. The rate at which the carriers become infected is represented by $\boldsymbol{\alpha}$. The rate at which the infected are taken care of at home is $\boldsymbol{\omega}$, if an individual is not taken care of at home, then they are hospitalized meaning the hospital representation is (1-$\boldsymbol{\omega}$). The rate at which the hospitalized recover is $\boldsymbol{\sigma}$ while the rate at which those on home-based care recover is τ. The rate at which the disease causes death is $\boldsymbol{\delta}$ while the rate of death which do not result from the disease or death through natural causes is represented by μ.

The table below indicates the summary of the parameters with their values and sources.

Table 1: Summary of parameter descriptions

Parameter	Interpretation	Value	Source
Λ	Recruitment rate to the population.	200	Assumed
β	Rate of recruitment to infectious from susceptible	0.0002	Mohammad A K *et al.*,
λ	Rate of recruitment of carriers from susceptible	0.00005	Estimated
α	Rate of recruitment of carriers to infectious	0.01	Estimated
δ	Death rate as a result of typhoid	0.002	Mohammad *et al.*,

	fever.		2015
ϕ	Recovery Rate	0.8	Joyce Nthiiri *et al.*, 2016
ω	Home based care rate	0.7	Estimated
1- ω	Hospitalization rate	0.3	Estimated
τ	Recovery rate for home-based patients	0.9	Estimated
σ	Recovery rate of hospitalized care patients	0.8	Estimated
μ	Death rate due to natural calamities.	0.0143	Stephen Edward

3.3 Model Equations

Dynamic system and differential equations.

Dynamic systems are set of equations which describes an event in nature that further describes primarily a time changing process. The properties which characterize these dynamical equations are either finite or infinite dimensions or being non-deterministic or deterministic in nature. The description of these systems is by use of differential equations.

Differential equations are defined as equations which contain a single or more derivatives which are of unknown functions.

The differential equations below are obtained from the model.

$$\frac{dS}{dt} = \Lambda + \phi R - \mu S - \beta SI - \lambda SC \quad 3.3.1$$

$$\frac{dC}{dt} = \lambda SC - (\alpha + \mu)C \quad 3.3.2$$

$$\frac{dI}{dt} = \beta SI + \alpha C - \big(\mu + \delta + \omega + (1-\omega)\big)\mathrm{I} \quad 3.3.3$$

$$\frac{dH}{dt} = (1-\omega)I - (\mu + \sigma + \delta)H \quad 3.3.4$$

$$\frac{dH_b}{dt} = \omega I - (\mu + \tau + \delta)H_b \quad 3.3.5$$

$$\frac{dR}{dt} = \sigma H + \tau H_b - (\mu + \phi)R \quad 3.3.6$$

3.4 The Invariant Region

This is the region which the model solution lies positively. We took into account all the human population (N), in which $N = S + C + I + R + H + H_b$. Differentiating N with respect to time (t), we obtained;

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dC}{dt} + \frac{dI}{dt} + \frac{dH}{dt} + \frac{dH_b}{dt} + \frac{dR}{dt} \quad (3.4.7)$$

By combining 4.0.1 and 5.1.1 we obtained

$$\frac{dN}{dt} = -\mu N + \Lambda - (I + H + H_b)\delta \quad (3.4.8)$$

In absentia of death due to typhoid fever disease, $\delta = 0$ equation (5.1.2) becomes

$$\frac{dN}{dt} = \Lambda - \mu N \quad (3.4.9)$$

Integrating both sides of equation (5.1.3) we obtain;

$$\int \frac{dN}{-\mu N + \Lambda} \leq \int dt$$

$$-\frac{1}{\mu}\ln(-\mu N + \Lambda) \leq t + C \quad (3.4.10)$$

Which then simplifies to;

$$-\mu N + \Lambda \geq Ae^{(-\mu t)} \quad (3.4.11)$$

Applying the initial condition when A is constant $N(0) = N_0$ in (5.1.5) yields

$$-\mu N + \Lambda \geq (-\mu N_0 + \Lambda)e^{(-\mu t)} \quad (3.4.12)$$

Then by rearranging (5.1.6) we obtain;

$$N \geq \frac{\Lambda}{\mu} - \left(\frac{\Lambda - \mu N_0}{\mu}\right) e^{-\mu t} \quad (3.4.13)$$

As t tends to infinity, that is $t \to \infty$ in equation (5.1.7), the population size $N \to \frac{\Lambda}{\mu}$ which means that $0 \leq N \leq \frac{\Lambda}{\mu}$. Thus implying that the feasible set of solution in the model remains and enters in the region.

$$\Omega = \left\{(S, C, I, Hb, H, R) \in R \ : N \leq \frac{\Lambda}{\mu}\right\} \quad (3.4.14)$$

This means that it is positively invariant and bounded.

3.5 Basic Reproduction Number

The basic reproductive number refers to the mean secondary infections which are caused by an infected individual who is able to transmit the disease over their entire time of being infectious. In the study of diseases, the basic reproduction number sets the pace or threshold in predicting the nature of the disease or its outbreak and evaluates possible control strategies. The persistence or the end of a disease is dependent on the basic reproductive value. The basic reproduction value is further used in analysis of equilibrium stability. If the basic reproduction value is less than one, this implies that an infectious individual causes less than a single secondary infection causing the disease to die out naturally. When the reproductive number is greater than unity, it means that an infectious individual will cause will cause more than one infections meaning that there will be an invasion of the disease in the population. A major pandemic may occur if the reproduction number is large.

In this thesis, the mean number of new typhoid infections is accounted by the reproduction number in which a typhoid infected individual gets introduced to a fully susceptible population.

The basic reproduction number is computed by use of the next generation matrix approach. It is mostly denoted by R_0 which is the mean number of secondary infections when an infected enters a susceptible population. The method of obtaining the reproduction number is worked out below.

$$Matrix \quad G = FV^{-1} \qquad (3.5.15)$$

We let X to be the vector of class which is infected, which are carriers, infectious, home-based care and hospitalized. We let Y be the vector of uninfected classes that is susceptible and recovered.

$$X = \begin{bmatrix} C \\ I \\ H \\ H_b \end{bmatrix} \quad and \quad Y = \begin{bmatrix} S \\ R \\ \end{bmatrix}$$

$F(X, Y)$ becomes the vector containing new infection rates.

$V(X, Y)$ is the vector of all other rates not new infections.

$$F = \begin{Bmatrix} \lambda SC \\ \beta SI \\ 0 \\ 0 \end{Bmatrix} \qquad (3.5.16)$$

$$V = \begin{Bmatrix} (\mu + \alpha)C \\ -\alpha C + (\mu + \delta + (1 - \omega) + \omega)I \\ -\omega I + (\mu + \tau + \delta)H_b \\ (1 - \omega)I + (\mu + \sigma + \delta)H_b) \end{Bmatrix} \qquad (3.5.17)$$

Calculating the jacobian of F and V becomes

$$F = \begin{pmatrix} \frac{\partial F^1}{\partial C} & \frac{\partial F^1}{\partial I} & \frac{\partial F^1}{\partial H} & \frac{\partial F^1}{\partial H_b} \\ \frac{\partial F^2}{\partial C} & \frac{\partial F^2}{\partial I} & \frac{\partial F^2}{\partial H} & \frac{\partial F^2}{\partial H_b} \\ \frac{\partial F^3}{\partial C} & \frac{\partial F^3}{\partial I} & \frac{\partial F^3}{\partial H} & \frac{\partial F^3}{\partial H_b} \\ \frac{\partial F^4}{\partial C} & \frac{\partial F^4}{\partial I} & \frac{\partial F^4}{\partial H} & \frac{\partial F^4}{\partial H_b} \end{pmatrix} = \begin{pmatrix} \lambda S & 0 & 0 & 0 \\ 0 & \beta S & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad (3.5.18)$$

$$V = \begin{pmatrix} \frac{\partial V^1}{\partial C} & \frac{\partial V^1}{\partial I} & \frac{\partial V^1}{\partial H} & \frac{\partial V^1}{\partial H_b} \\ \frac{\partial V^2}{\partial C} & \frac{\partial V^2}{\partial I} & \frac{\partial V^2}{\partial H} & \frac{\partial V^2}{\partial H_b} \\ \frac{\partial V^3}{\partial C} & \frac{\partial V^3}{\partial I} & \frac{\partial V^3}{\partial H} & \frac{\partial V^3}{\partial H_b} \\ \frac{\partial V^4}{\partial C} & \frac{\partial V^4}{\partial I} & \frac{\partial V^4}{\partial H} & \frac{\partial V^4}{\partial H_b} \end{pmatrix}$$

$$V = \begin{pmatrix} (\mu+\alpha) & 0 & 0 & 0 \\ -\alpha & (\mu+\delta+\omega+(1-\omega)) & 0 & 0 \\ 0 & -\omega & (\mu+\tau+\delta) & 0 \\ 0 & -(1-\omega) & 0 & (\mu+\sigma+\delta) \end{pmatrix}$$

....(3.5.19)

Obtaining V^{-1} becomes;

$$V^{-1} =$$

$$\begin{bmatrix} (\alpha+\mu)^{-1} & 0 & 0 & 0 \\ \frac{\alpha}{(\alpha+\mu)(\mu+1+\delta)} & (\mu+1+\delta)^{-1} & 0 & 0 \\ \frac{\omega\,\alpha}{(\alpha+\mu)(\mu+1+\delta)(\mu+\delta+\tau)} & \frac{\omega}{(\mu+1+\delta)(\mu+\delta+\tau)} & (\mu+\delta+\tau)^{-1} & 0 \\ -\frac{(-1+\omega)\alpha}{(\alpha+\mu)(\mu+1+\delta)(\delta+\mu+\sigma)} & -\frac{-1+\omega}{(\mu+1+\delta)(\delta+\mu+\sigma)} & 0 & (\delta+\mu+\sigma)^{-1} \end{bmatrix}$$

(3.5.20)

$$FV^{-1} = \begin{bmatrix} \frac{\lambda\,\Lambda}{\mu\,(\alpha+\mu)} & 0 & 0 & 0 \\ \frac{\beta\,\Lambda\,\alpha}{\mu\,(\alpha+\mu)(\mu+1+\delta)} & \frac{\beta\,\Lambda}{\mu\,(\mu+1+\delta)} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ ....(3.5.21)

The eigen values are given by

$$\begin{bmatrix} 0 \\ 0 \\ \frac{\beta \Lambda}{\mu (\mu+1+\delta)} \\ \frac{\lambda \Lambda}{\mu (\alpha+\mu)} \end{bmatrix} \quad \text{.......(3.5.22)}$$

The most dominant eigen value gives the basic reproduction number R_0. Therefore

$$R_0 = \frac{\beta \Lambda}{\mu (\mu+1+\delta)} \quad \text{.......(3.5.23)}$$

3.6 Disease Free Equilibrium

In disease free equilibrium, we qualitatively analyze the stability of its equilibrium. The disease free equilibrium points of the model at its steady state in the absence of disease.

To obtain equilibrium points we let

$$\frac{dHb}{dt} = \frac{dC}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = \frac{dH}{dt} = 0, \quad \text{.......(3.6.24)}$$

$$\frac{dS}{dt} \neq 0 \text{ This implies } \frac{dHb}{dt} = \frac{dC}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = \frac{dH}{dt} = 0 \quad \text{.......(3.6.25)}$$

hence no differential. By setting the differential equations to be zero, we obtain;

$$\begin{aligned}
&\Lambda + \phi R - \mu S - \beta SI - \lambda SC = 0 \\
&\lambda SC - (\alpha + \mu)C = 0 \\
&\beta SI + \alpha C - \big(\mu + \delta + \omega + (1 - \omega)\big)\mathrm{I} = 0 \\
&(1 - \omega)I - (\mu + \sigma + \delta)H = 0 \\
&\omega I - (\mu + \tau + \delta)H_b = 0 \\
&\sigma H + \tau H_b - (\mu + \phi)R = 0
\end{aligned}$$

We assume that there is no disease, therefore, when $C = 0$, $I = 0$, $H_b = 0$ $H = 0$ and $R = 0$. $S = N$ but $S \neq 0$

$$\Lambda + \phi R - \mu S - \beta SI - \lambda SC = 0$$

We obtain; $\Lambda - \mu S = 0$. ....(3.6.26)

Making S the subject of the formula, below is obtained;

$$S = \frac{\Lambda}{\mu} \quad \text{...(3.6.27)}$$

Hence $D.F.E = (S^*, C^*, I^*, Hb^*, H^*, R^*)$

$$D.F.E = \left(\frac{\Lambda}{\mu}, 0,0,0,0,0\right)$$

3.7 Local Stability of Disease Free Equilibrium

We analysed qualitatively the stability of disease free equilibrium that is the absence of disease. From the model system, we have jacobian matrix at disease free equilibrium of the linearized system given by;

$$J = \begin{bmatrix} -\mu & -\frac{\lambda\Lambda}{\mu} & -\frac{\beta\Lambda}{\mu} & 0 & 0 & \phi \\ 0 & \frac{\lambda\Lambda}{\mu} - \alpha - \mu & 0 & 0 & 0 & 0 \\ 0 & \alpha & \frac{\beta\Lambda}{\mu} - 1 - \delta - \mu & 0 & 0 & 0 \\ 0 & 0 & 1-\omega & -\delta-\mu-\sigma & 0 & 0 \\ 0 & 0 & \omega & 0 & -\mu-\delta-\tau & 0 \\ 0 & 0 & 0 & \sigma & \tau & -\mu-\phi \end{bmatrix}$$

....(3.7.28)

which yields the following eigen values:

$$\varepsilon = \begin{bmatrix} -\mu \\ -\mu - \delta - \tau \\ -\delta - \mu - \sigma \\ -\mu - \phi \\ \frac{\beta\,\Lambda - \delta\,\mu - \mu^2 - \mu}{\mu} \\ \frac{\lambda\,\Lambda - \alpha\,\mu - \mu^2}{\mu} \end{bmatrix} \text{..........} (3.7.29)$$

The first four eigen values are negative therefore to make the sytem stable we need to have

$$\frac{\beta\,\Lambda - \delta\,\mu - \mu^2 - \mu}{\mu} > 0 \text{..........} (3.7.30)$$

therefore

$$\frac{\beta\,\Lambda}{\mu} > \delta + \mu + 1$$

again

$$\frac{\lambda\,\Lambda - \alpha\,\mu - \mu^2}{\mu} > 0$$

therefore

$$\frac{\lambda\,\Lambda}{\mu} > \alpha + \mu \text{..........} (3.7.31)$$

In conclusion, if $\frac{\lambda\,\Lambda}{\mu} > \alpha + \mu$ and $\frac{\beta\,\Lambda}{\mu} > \delta + \mu + 1$ this means the disease free equilibrium is asymptotically locally stable.

CHAPTER FOUR

RESULTS AND DISCUSSIONS

4.1 The Invariant Region

The total population N is the sum of the population in the susceptible, carriers, infected, home-based care, hospitalization and recovered i.e. N=S+C+I+R+H+Hb then $0 \leq N \leq \frac{\Lambda}{\mu}$; this shows that the total population (N) is greater than zero which is a proof that the model solution lies positively and is bounded.

4.2 The Basic Reproduction Number

The basic reproduction number is an estimation which determines if there will be an outbreak of the disease or not.

If R0<1 then an individual cause less than one secondary infection therefore the disease dies out.

If R0>1 means an individual cause more than one secondary infection therefore the disease invades the population.

Since the basic reproduction number is estimation, the most dominant eigen value is picked which is $R_0 = \frac{\beta\,\Lambda}{\mu\,(\mu+1+\delta)}$ from equation ....(3.5.23)

When **β**=0.0002

μ=0.0143

$\boldsymbol{\delta}$ =0.002

Λ=200

Substituting these values R0=2.7523, therefore R0 > 1 which means that the disease invades the population and persists. The reproduction number is close to one, therefore a pandemic may not occur.

4.3 The Disease Free Equilibrium

The estimation of the basic reproduction number determines the disease free equilibrium. At DFE, the determinant of the Jacobean matrix is positive at R0 > 1 then the model is stable.

Hence $D.F.E = (S^*, C^*, I^*, Hb^*, H^*, R^*)$

$$D.F.E = \left(\frac{\Lambda}{\mu}, 0,0,0,0,0\right)$$

$S = \frac{\Lambda}{\mu}$ The susceptible population is the total population which is free of the disease while C=I=Hb=H=R=0, this means that the carriers, the infected, home-based care and hospitalized are not there because there is no disease in the equilibrium. Since there is no disease, no one recovers, therefore R=0

4.4 Local Stability of the Disease Free Equilibrium

The equation $\frac{\lambda \Lambda}{\mu} > \alpha + \mu$ is true,

Proof: λ=0.00005

Λ=200

μ=0.0143

$\boldsymbol{\alpha}$=0.01

Replacing the parameters with the values, 0.6993>0.0243 is obtained.

The equation $\frac{\beta \Lambda}{\mu} > \delta + \mu + 1$ is true,

Proof:

Λ=200

μ=0.0143

β=0.0002

$\boldsymbol{\delta}$=0.002

Replacing the parameters with the values, 2.797>1.063 is obtained.

This is the proof that the disease free equilibrium is asymptomatically locally stable.

4.5 Graphical Solutions

4.5.1 Representation of the Dynamical System

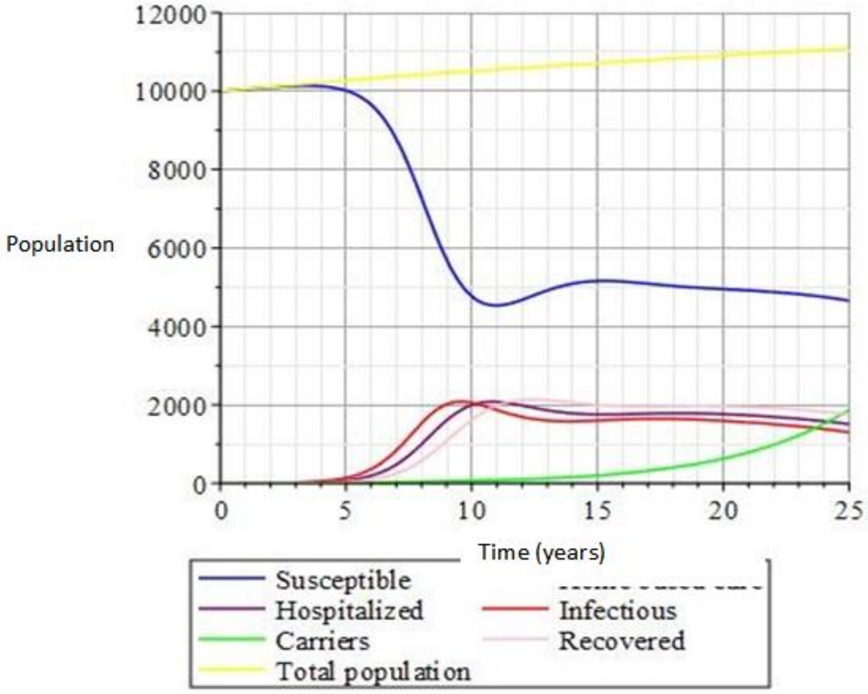

Figure 4.1 Graphical Representation of the Dynamical System

The total population (N) is approximated at 10000 individuals in a location. The recruitment rate is 200 people per year, that is, mainly from births and to a lesser extent immigration. The recruitment rate would have been higher if the death rate was not considered; the recruitment rate is arrived at by taking an inclusion of immigration rate, birth rate and death rate then working out the average rate. This explains the constant rise of the total population in the dynamical system. The total population is assumed that at the initial year all the human population is susceptible to the typhoid fever disease; this implies that all individuals are likely to be affected by the disease. When an infectious disease enters a susceptible population, the susceptible population tends to decrease with increasing infectious population. When infections rise in a population, the population of the carriers increases with time leading to an increased widespread of the infectious disease, this is attributed to the fact that carriers are asymptomatic. An increase in infections leads to the sick individuals being taken care of at homes. Similarly, an increase in infections implies an increase in hospitalization of patients.

4.5.2 Clustered populations

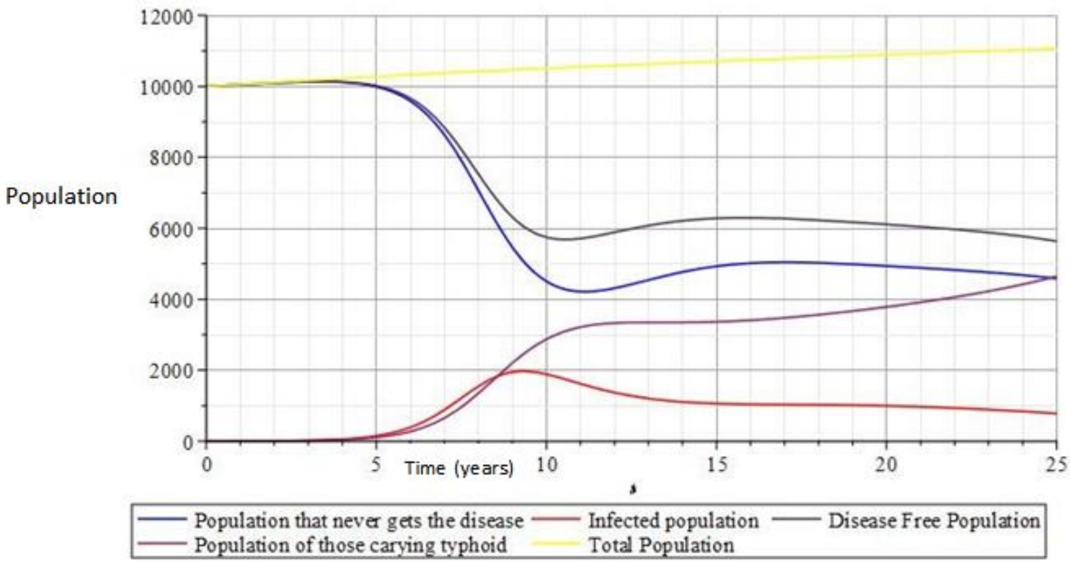

Figure 4.2 clustered populations

Similar to Figure 4.1, an initial population of 10000 individuals is taken. The recruitment rate of humans to a clustered population is Λ. The rate is arrived at by averaging the approximate death mortality rate, the approximate birthrate and the approximate immigration rate to obtain the average recruitment rate (Λ) of 200 humans per year. The whole human population at the initial time is assumed to be susceptible which means that all individuals are likely to be infected by the typhoid fever disease. Susceptibility reduces with increasing typhoid infections. However, there is no time when all humans lose their susceptibility to the disease, this is majorly caused by those who recover and attain new susceptibility. The disease free population is an individual who do not have the typhoid fever or are free of the typhoid fever. At the initial state, the whole population is devoid of the disease. With new infections, the disease free population declines with increase in infections. However, the decline does not lead to the whole population being infected, this is due to those who recover from the disease hence not all the population will be infected. An increase in typhoid fever infections tends to lead to an increase in the carrier population.

4.5.3 Infectious population at different rates of hospitalization.

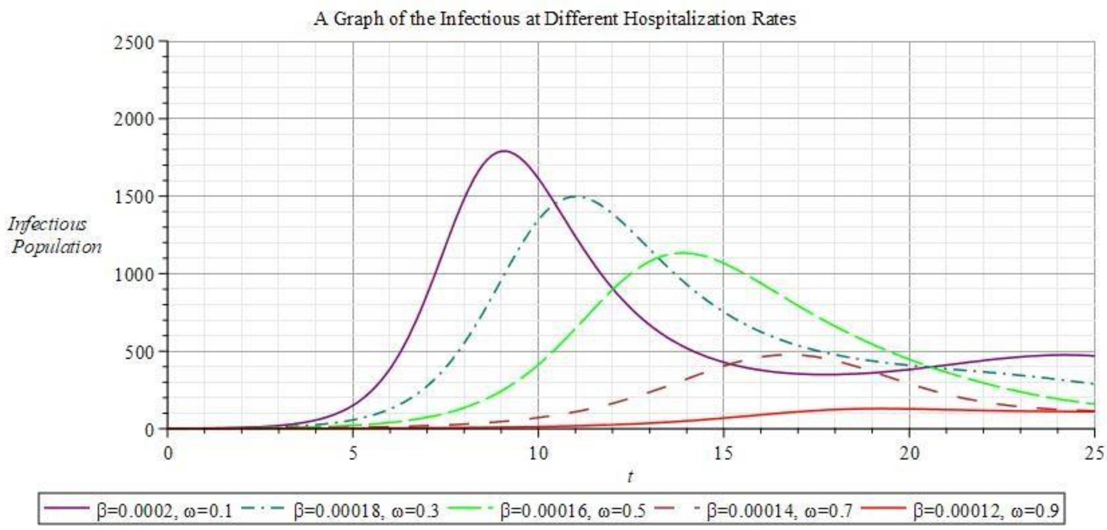

Figure 4.3: Graph of Infectious population at different rates of hospitalization

From figure 4.1 and figure 4.2, the infectious population is approximately 2500 individuals. Hospitalization is the process in which infected individuals are taken care of at a medical facility by medical practitioners. The rate of hospitalization or those infected by the disease are taken for medication is ω while the rate of infections is β. When the rate of new infections per year is high and less individuals being hospitalized, the infectious population is very high. In the case when β=0.0002 and ω=0.1, the infectious population gets to its peak within a short period of time. Managing such cases can pose challenges to the health sector since they can get overwhelmed with the disease because of high infections in an increasing population. At the rate when **β**=0.00018 and ***ω***=0.3, this implies that the rate of hospital management is increased, there would be a decline in the number of infections. The infectious population gets to the peak after a long time compared to the first rate and the infectious population becomes lower. At the rate when **β**=0.00016 and ***ω***=0.5, this means that with the rate of hospital management increasing, the rate of infections decrease. The infectious population at this point lowers at its peak in a longer period of time. At the rate when **β**=0.00014 and ***ω***=0.7, this shows an increase in hospital management of the disease being higher while the rates of infection decline. An increase in hospital management leads to a decline in infection rates. The infectious population will have been decreased considerably and can be managed with ease even at its peak. At the rate of **β**=0.00012 and ***ω***=0.9, this is a clear implication that the larger the hospitalization rate the lesser the number of infections. The graph in this case is steady meaning the disease has been contained and poses no risk to human life.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

5.1.1 Dynamical system

There is increase in the total population with time in the dynamical system. Naturally, in a community set-up, there tend to be an increase in the total population due to the birth rate and immigration rate. Despite the fatalities caused by the disease or natural calamities, the population will still rise. The susceptible population is equal to the total population at the beginning of the first year. When the typhoid fever infections begin, the susceptible population drops drastically since most susceptible individuals will have been infected and others will become carriers of the typhoid fever disease. This concludes that an increase in the rate of infection leads to a decrease in susceptible population. The hospitalized and the home-based care individuals are responsible for the decline in the number of infections. This is clear in that the number of recoveries increases and then attains secondary susceptibility to the disease. The carrier population rises consistently since they are asymptomatic; therefore, it poses a risk since they transmit the disease unknowingly. The diagnosis of the carrier population is challenging therefore providing treatment to such individuals can be difficult. Hospitalization aids mainly in reducing the infections or the infectious rates of individuals with the typhoid fever disease.

5.1.2 clustered populations.

There is increase in the total population with time in the dynamical system. Naturally, in a community set-up, there tend to be an increase in the total population due to the birth rate and immigration rate. Despite the fatalities caused by the disease or natural calamities, the population will still rise. The susceptible population is equal to the total population at the beginning of the first year, susceptibility of individuals' drops with new infections of typhoid fever. With increased infections, the susceptible population reduces in number therefore susceptibility is inversely proportional to the number of infections. The disease free population is similar to the susceptible population; this is because at the beginning, the total population is equal to the disease free population. When infections rise in a population, those who are devoid of the disease tend to reduce. This implies that an increase in the infected population results to a decline in the population of those without the disease. Recoveries also contribute to an increase of those individuals without the disease. The population without the disease is more compared to those susceptible, since an individual may not have the disease but is not susceptible. The carrier population continuously increases with time hence controlling the carrier population becomes a challenge. The asymptomatic nature of the carrier population results to increased rates of infections. In general, an increase in infection results to a decline in the susceptible and disease free population classes.

5.1.3 Infectious Populations at Different Rates of Hospitalization

Hospital management of typhoid fever disease patients plays a major role in the control of typhoid fever infections. When the rate of hospital management or hospitalization is very low, the infectious population is high meaning that controlling the infected population can be tasking. A requirement in improving the health sector by increasing the number of hospitals as well as increasing the bed capacity is essential in the management of the disease. This implies that an increase in the number of hospitals will require an increase in the health care providers. Figure 4.3 shows that increasing the hospital rate tends to decrease the infectious rate of the disease. The graph clearly shows that increasing hospitalization leads to a decline in the number of infected individuals hence this reduces the number of infections in which an individual can transmit. When the rate of hospital management is low, the rate of infections can be very high, this implies that the number an infected individual can transmit within the period of infection can be very high in a population. In conclusion, the rate of hospitalization is inversely proportional to the infectious population.

5.2 Recommendations

The mathematical model focuses on the importance of hospitalization in the management of typhoid fever through its treatment. However, from the graphical analysis there is a risk in the rising case of carriers with time and do not drop or stabilizes with the constantly increasing population. This implies that attention need tro be given more on curbing the increasing number of carriers in the population.

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APPENDICES

Appendix I: Similarity Report

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