OVERALL CENTRAL MOMENTS OF TRAFFIC DELAY AT A SIGNALIZED INTERSECTION

RONOH KIPROTICH BENARD

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DECLARATION

Declaration by the student

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Signature …………………….. **Date ……………………**

Ronoh Kiprotich Benard

(SC/PGM/013/10)

Declaration by the supervisors

This thesis has been submitted for examination with our approval as the university supervisors.

Signature …………………….. **Date ……………………**

Prof. Nyongesa Kennedy,

Department of Mathematics,

Masinde Muliro University of Science & Technology,

P.O Box 190-00560,

Kakamega, Kenya.

Signature …………………….. **Date ……………………**

Dr. Otieno Argwings,

Department of Mathematics and Computer Science,

University of Eldoret,

P.O Box 1125,

Eldoret, Kenya.

DEDICATION

To my parents and my wife Janeth for their strengths and encouragements.

ABSTRACT

Overall traffic delay model for estimating the mean and variance at a signalized intersection is discussed. The model was developed under basis of two delay components, namely deterministic and stochastic components. The latter component was put under D/D/1 framework and therein mean and its variance derived. While the stochastic component was put under the M/G/1 framework, mean and variance derived. Extension on stochastic component and M/G/1 framework was discussed with the usage of compressed queueing processes. Harmonization of the moments of deterministic and stochastic components to obtain the overall central moments of traffic delay has been discussed. Illustration of the model on real traffic data has been carried out. Simulation was performed using statistical software for traffic intensities ranging from 0.1 to 1.9. The simulated results indicate that both deterministic and stochastic components are incompatible as the traffic intensity approaches capacity. Also, the simulation shows that variance of overall traffic delay drops linearly when the traffic intensity is less than 3 because of increasing rate of random arrivals. This variance decreases slowly as the traffic intensity approaches capacity and slowly increases as the traffic intensity goes beyond capacity. This confirms the results that exist in literature that oversaturated conditions and random delay renders the stochastic component in traffic delay models unrealistic.

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LIST OF SYMBOLS AND NOTATIONS

Q_o - Expected overflow queue

c_a - Capacity (veh/h)

c_y - Cycle time (sec)

l_1 - Start-up lost time (sec)

l_2 - Clearance lost time (sec)

t_L - Total lost time for a movement during a cycle (sec)

t - Evaluation period (sec)

ρ - Traffic intensity

AR - Displayed all-red time (sec)

G - Displayed green time (sec)

I - Index of dispersion for the arrival process.

R - Displayed red time (sec)

Y - Displayed yellow time (sec)

g_e - Effective green time (sec)

r - Effective red time (sec)

s - Saturation flow rate (veh/h)

h - Height of the cross-sectional area ABC in Figure (4.1)

iid - Independent and identically distributed

d - Average delay per vehicle (sec)

λ - Traffic arrival rate

μ - Departure flow rate from queue during green time

B^2 - Index of dispersion for the departure process

W_1 - Total delay experienced in the red phase (sec)

W_2 - Total delay experienced in the green phase (sec)

μ' - Service time rate for a compressed model

λ' - Arrival rate for the compressed model

g_o - Time necessary for the queue to dissipate

LoS – Level of service

D_t - Overall traffic delay

D_{t_1} - Deterministic delay component of traffic delay

D_{t_2} - Stochastic delay component of traffic delay

Q_t - Number of vehicles waiting in the queue

R_t - Residual time for vehicle j (time until the vehicle found being served completes the service)

W_t - Waiting time for vehicle j

X_t - Service time for vehicle j

$E\ D_t$ - Expected deterministic delay component

$E\left[D_{t_1}\right]$ - Expected stochastic delay component

$E\left[D_{t_2}\right]$ - Expected overall traffic delay

$Var\ D_t$ - Variance of deterministic delay component

$Var\left[D_{t_1}\right]$ - Variance of stochastic delay component

$Var\left[D_{t_2}\right]$ - Variance of overall traffic delay

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CHAPTER ONE

INTRODUCTION

1.1 Background Information

Traffic delays and queues are principal measures of performance that determine the level of service (LoS) at signalized intersections. They also evaluate the adequacy of the lane lengths and the estimation of fuel consumption and emissions. Quantifying these delays accurately at an intersection is critical for planning, design and analysis of traffic lights. Signalized intersection referred herein, is a road junction controlled by a traffic light. Traffic lights were implemented for the purpose of reducing or eliminating congestions at intersections. These congestions exist because an intersection is an area shared among multiple traffic streams, and the role of the traffic light is to manage the shared usage of the area. Traffic models in an intersection are always subjected to both uniform and random properties of traffic flows. As a result of these properties, vehicle travel times in an urban traffic environment are highly time dependant.

Models that incorporate both deterministic and stochastic components of traffic performance are very appealing in the signalized intersection since they are applied in a wide range of traffic intensities as well as to various types of traffic lights. They simplify theoretical models with delay terms that are numerically inconsequential. Due to their simplicity, the models have been incorporated in many intersection traffic lights and as tools for analysis of intersections on roads throughout the world. The theory behind the uniform and random properties of traffic flows is based on the works of Webster (1958). For instance, the problems of estimating delays at signalized intersections have been extensively studied in the literature; however, majority of the works have focused on developing models for estimating mean delay only.

At an intersection where certain approaches are denied movement, queueing will inherently occur resulting to traffic delay models based on the queueing theory being developed. Of the various queueing models, D/D/1 and $M_{+\Delta}/G_{+\Delta}$ /1 were used in this study. D implies a degenerate distribution (constant time) of inter-arrival and service times, M implies exponential distribution of inter-arrival times, G implies general distribution (any arbitrary distribution), Δ implies the time distance between vehicles at the queue and 1 implies one server (traffic light). The D/D/1 model assumed that the arrivals and departures were uniform and one service channel (traffic light) existed. This model is quite intuitive and easily solvable. Using this form of queueing with an arrival rate, denoted by λ and a service rate, denoted by μ, certain useful values regarding the consequences of queues were computed. The $M_{+\Delta}/G_{+\Delta}$ /1 model used implied that the vehicles arrived at an intersection in a Poisson process with rate λ and were treated in the order of arrival with inter arrival times following exponential distribution with parameter μ. The service times were treated as independent identically distributed with an arbitrary distribution. Similarly, one service channel (traffic light) was considered in this model. This thesis is structured as follows: In Chapter two, we provide the literature review of the study. In Chapter three, we introduce the methods applied in this study. The core of this work is described in Chapter four, where we show how we apply the methods to develop the overall traffic delay model. Chapter five presents simulation results and discussions. Chapter six summarizes the work presented and gives possible future work in this area.

1.2 Statement of the Problem

Traffic delays at signalized intersections are becoming a nuisance on the Kenyan roads. For instance, one of the things that leaves a mark on visitors who tour Kenya's capital city Nairobi, is its chronic traffic delays which could last for several hours. The worst nightmare is that these delays are experienced during peak times and heavy downpour. During working days, in the morning and evening, 70% of Nairobi's work force is held up at an intersection due to traffic delays. As a result, the economy of the country is estimated to be about 1.5 billion shillings in lost man-hours and fuel, Wilfred (2011). The losses incurred are not only confined to fuel consumption but also to environmental pollution and stress. The contributor in most cases is as a result of fixed-time traffic lights with uniform arrival and service times. This study develops the overall traffic delay model using D/D/1 and compressed $M_{+\Delta}/G_{+\Delta}$ /1 queueing systems. D/D/1 implies inter-arrival and service times are deterministic while $M_{+\Delta}/G_{+\Delta}$ /1 implies Markovian arrivals and iid service times following a general distribution. 1 in these systems represents a single service channel (traffic light).

1.3 Main Objective

The main objective of this study was to develop overall traffic delay model for estimating the mean of the time delay and its variance at a signalized intersection.

1.3.1 Specific Objectives

The specific objectives of the study are

i. To develop models estimating mean and variance of both deterministic and stochastic delay components;
ii. To develop the mean and variance of the overall traffic delay model;
iii. To apply the model on real traffic data.

1.4 Significance of the Study

This model when implemented can help in easing up the traffic delay at a signalized intersection.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This chapter provides the discussion of the literature behind the traffic delay models and its surveys. The literature is split into deterministic delay models, steady-state delay models, time dependent models and application of compressed queueing processes. Some mathematical models are also presented as they existed in the literature. The chapter is summarized as follows: Section 2.2 provides literature on deterministic delay models while steady-state and time dependent models are discussed in Section 2.3 and 2.4, respectively. Section 2.5 discusses the application of compressed queueing processes.

2.2 Deterministic Delay Models

Zukerman (2012) considered a case where the inter-arrival and service times are deterministic. To avoid ambiguity, he assumed that if an arrival and a departure occur at the same time, the departure occurs first. According to him, such an assumption is not required for Markovian queues where the queue size process follows a continuous-time Markov-chain because the probability of two events occurring at the same time is zero, but it is needed for deterministic queues. Unlike many of the Markovian queues, steady-state queue size distribution for the deterministic queues does not exist because the queue size deterministically fluctuates according to a certain pattern. According to him, the mean queue size, denoted by $E[Q]$, is given by

$$E[Q] = \sum_{n\geq 0}^{\infty} n \Pr[Q = n], \tag{2.1}$$

where $\Pr(Q=n)$ is the probability of having n vehicles in the queue at a randomly chosen point in time. As all the vehicles that enter the system are served before the next one arrives, the mean queue-size of D/D/1 must be equal to the mean queue-size at the traffic light, and therefore, it is also equal to the traffic intensity. In other words, the queue-size alternates between the values 1 and 0, spending a time-period of $1/\mu$ at state 1, then a time-period of $1/\lambda - 1/\mu$ at state 0, then again $1/\mu$ time at state 1, etc. If we pick a random point in time, the probability that there is one in the queue is given by $P(Q=1) = (1/\mu) / (1/\lambda)$, and the probability that there are no vehicles in the queue is given by $P(Q=0) = 1 - (1/\mu) / (1/\lambda)$. Therefore, (2.1) becomes,

$$E(Q) = 0.P(Q=0) + 1.P(Q=1) = (1/\mu) / (1/\lambda) = \lambda/\mu = \rho. \quad (2.2)$$

2.4 Steady-State Models

These models characterize traffic delays based on statistical distributions of the arrival and departure processes. Because of the purely theoretical foundation of the models, they require very strong assumptions to be considered valid. The following section describes the exact expressions on how steady-state delays are estimated.

2.4.1 Exact Expressions

Beckman (1956) derived the expected delay at fixed-time signals with the assumption of the binomial arrival process and deterministic service given by

$$d = \frac{c_y - g_e}{c_y\left(1 - \frac{\lambda}{\mu}\right)}\left(\frac{Q_o}{\lambda} + \frac{c_y - g_e + 1}{2}\right), \quad (2.3)$$

where d, c_y, g_e, λ, μ and Q_0 are provided in the list of symbols. The expected overflow queue used in the formula and the restrictive assumption of the binomial arrival process reduce the practical usefulness of (2.3). Little (1961) analyzed the expected delay at or near traffic signals to a vehicle crossing a Poisson traffic lane. McNeil (1968) derived a formula for the expected signal delay with the assumption of a general arrival process, and constant departure time. From this work, Tarko at al. (1993) expressed the total vehicle delay during one signal cycle as a sum of two delay components

$$W = W_1 + W_2, \tag{2.4}$$

where W_1 and W_2 are provided in the list of symbols. With departure process being deterministic, Darroch (1964) took the expectations of W_1 and W_2 and obtained the expected vehicle delay as

$$d = \frac{(c_y - g_e)}{2c_y(1-\rho)}\left((c_y - g_e) + 2\frac{Q_o}{\lambda} + \frac{1}{\mu}\left(1 + \frac{I}{1-\rho}\right)\right), \tag{2.5}$$

where I is provided in the list of symbols. Equation (2.5) becomes identical to that obtained by Beckmann (1956) when arrival process follows a binomial distribution. Gazis (1974) considered the case of the compound Poisson arrival process and general departure process obtaining the following model

$$d = \frac{(c_y - g_e)}{2c_y(1-\rho)}\left((c_y - g_e) + 2\frac{Q_o}{\lambda}\left(1 + \frac{(1-\rho)(1-B^2)}{2\mu}\right) + \frac{1}{\mu}\left(1 + \frac{I + B^2\rho}{1-\rho}\right)\right), \tag{2.6}$$

where B^2 is provided in the list of symbols. Equation (2.6) indicates that in the case of no overflow queue $(Q_o = 0)$, and no randomness in the traffic process (I = 0), the resultant delay becomes the deterministic delay component. Section 2.4.2 discusses the approximate expressions on how steady-state delays are estimated.

2.4.2 Approximate Expressions

The numerical inconsequentiality in obtaining exact expressions for delay which are reasonably simple and can cover a variety of real world conditions, gave impetus to a broad effort for traffic delay estimation using approximate models and bounds. The first, widely used approximate delay formula which was developed by Webster (1958) from a combination of theoretical and numerical simulation approaches is

$$d = \frac{c_y\left(1-\frac{g_e}{c_y}\right)^2}{2\left(1-\frac{g_e}{c_y}\rho\right)} + \frac{\rho^2}{2\lambda(1-\rho)} - \left(0.65\left(\frac{c_y}{\lambda^2}\right)^{\frac{1}{3}} \times \rho^{\left(2+5\frac{g_e}{c_y}\right)}\right). \qquad (2.7)$$

The first term in (2.7) represents delay when traffic can be considered arriving at a uniform rate, while the second term makes some allowance for the random nature of the arrivals. The latter assumption does not reflect actual traffic performance, since vehicles are served only during the effective green time, obviously at a higher rate than the capacity rate. The third term in (2.7) which was calibrated basing on simulation experiments is a corrective term to the estimate.

Newell (1965) developed a delay formula for general arrival and departure distributions. He concluded from a heuristic graphical argument that for most reasonable arrival and departure processes, the total delay per cycle differs from that calculated with the assumption of uniform arrivals and fixed service times (Clayton,1941) by a negligible amount if the traffic intensity is sufficiently small. Then, by assuming LIFO (Last In First Out) queue discipline which does not affect the average delay estimate, he concluded that the expected delay when the traffic is sufficiently heavy can be approximated as

$$d = \frac{c_y\left(1-\frac{g_e}{c_y}\right)^2}{2\left(1-\rho\right)} + \frac{Q_o}{\lambda}. \tag{2.8}$$

To estimate Q_0, Newell (1965) defines F_Q as the cumulative distribution of the overflow queue length, F_{A-D} as the cumulative distribution of the overflow in the cycle, where the indices A and D represent cumulative arrivals and departures, respectively. He showed under equilibrium conditions that:

$$F_Q\left(x\right) = \int_0^\infty F_Q\left(z\right) dF_{A-D}\left(x-z\right). \tag{2.9}$$

The integral in (2.9) can be solved only under the restrictive assumption that the overflow queue in a cycle is normally distributed. Therefore, the expected overflow queue in (2.8) is given by

$$Q_o = \frac{\lambda c_y\left(1-\rho\right)}{\pi} \int_0^{\frac{\pi}{2}} \left(\frac{\tan^2\theta}{-1+e^{\left(\frac{\mu g_e\left(1-\rho\right)^2}{2\cos^2\theta}\right)}} \right) d\theta. \tag{2.10}$$

He further compared the results given by the expressions in (2.8) and (2.10) with Webster's formula (2.7) and added additional correction terms to improve the results for medium traffic intensity conditions. Thus his final formula became

$$d = \frac{c_y\left(1-\frac{g_e}{c_y}\right)^2}{2\left(1-\rho\right)} + \frac{Q_o}{\lambda} + \frac{\left(1-\frac{g_e}{c_y}\right)I}{2\mu\left(1-\rho\right)^2}. \tag{2.11}$$

2.5 Time Dependent Models

The stochastic equilibrium assumed in steady-state models requires an infinite time period of stable traffic conditions to be achieved. Traffic flows during peak hours are seldom stationary, thus violating an important assumption of steady-state models. Liping and Bruce (1999) developed a model for estimating arrival time dependent delay that is subject to large variation because of the randomness of traffic arrivals and interruption caused by traffic lights. The model was constructed on the basis of the delay evolution patterns under two extreme traffic conditions: highly undersaturated and highly oversaturated conditions. The model for estimating this delay was established through coordinate transformation based on the steady-state model and the deterministic model for arrival time dependent overflow delay (Kimber and Hollis, 1979). Liping and Bruce (1999) used the traditional uniform delay model and Canadian Capacity Guide (Teply et al., 1995) to estimate the mean arrival time dependent delay as

$$E(D) = \frac{c_y\left(1-\frac{g_e}{c_y}\right)^2}{2\left(1-\frac{g_e}{c_y}\rho_1\right)} + 0.5t\left((\rho_t - 1) + \sqrt{(\rho_t - 1)^2 + \frac{2\rho}{c_a t}}\right), \qquad (2.12)$$

where ρ_t is the traffic intensity at time t, ρ_1 is the minimum of $(1.0, \rho_t)$ and t represents the point in time (in seconds) for which arrival time dependent overflow delay is to be computed. For the case of variance, the variance of uniform delay was obtained theoretically on the basis of a deterministic queueing model (Rouphail 1995) while the variance of overflow queue was achieved by examining the relationship between the variances of the models obtained from the well-known Pollaczek-Khintchine formula for a M/G/1 system and the deterministic queueing theory. M/G/1 system assumed that the service times are independent identically distributed with mean $\frac{1}{\mu}$ and standard deviation σ_s while the arrival process is assumed to be Poisson with rate λ. Finally, Liping and Bruce (1999) described the variance of arrival time dependent delay as

$$Var\ \ D\ = \frac{c_y{}^2\left(1-\frac{g_e}{c_y}\right)^3 \times \left(1+3\frac{g_e}{c_y}-4\frac{g_e}{c_y}.\rho_1\right)}{12\left(1-\frac{g_e}{c_y}.\rho_1\right)^2}+\frac{t\rho}{c_a}e^{-\left(\frac{\rho_o}{\rho}\right)^{\beta}}\ , \tag{2.13}$$

where ρ_0 and β are the calibrated parameters determining the shape of the delay curve.

2.6 Compressed Queueing Processes

Grzegorz and Janusz (2007) derived a delay model comprising of deterministic model described by Clayton (1941) and expected waiting time from $M_{+\Delta}/G_{+\Delta}$ /1 queueing model with usage of the compressed queueing processes theory described by Woch (1998). The model is used to estimate mean delays in the case of large variations of the service time and has a form as follows

$$d = \frac{c_y\left(1-\frac{g_e}{c_y}\right)^2}{2\left(1-\frac{g_e}{c_y}\rho\right)}+\frac{\lambda\sigma_s{}^2+\lambda\left(\frac{1}{\mu}-\Delta\right)^2}{2\ \ 1-\rho}\times\ 1-\mu\Delta\ \ , \tag{2.14}$$

where Δ is provided in the list of symbols. Equation (2.14) makes a generalization of the Webster's model (1958). Webster used the steady-state M/D/1 queueing model to come up with (2.7). To reflect the real traffic situation, the methods employed are described in the next chapter.

CHAPTER THREE

METHODOLOGY

3.1 Introduction

This chapter presents the methods that were used in this study. Section 3.2 provides the use of D/D/1 queueing system while the use of M/G/1 queueing system is described in Section 3.3. Section 3.4 provides the use of compressed $M_{+\Delta}/G_{+\Delta}/1$ queueing system while statistical software for simulation is mentioned in Section 3.5. Section 3.6 describes the sampling method used to collect the traffic data at Kenyatta Avenue-Kimathi Street signalized intersection.

3.2 D/D/1 Queueing System

Overall traffic delay model can be split into two categories, that is, deterministic and stochastic delay components. To analyze the deterministic delay component, we employed the use of D/D/1 queueing system. This system is founded on the uniform property of traffic flows in which the inter-arrival and service times are deterministic, that is, the first D represents uniform arrivals with parameter λ, the second D representing constant departures with parameter μ and 1 representing one service channel (traffic light).

3.3 M/G/1 Queueing System

The stochastic delay component of the overall traffic delay can appropriately be analyzed using the framework of M/G/1 queueing system. This system is founded on the steady-state queueing theory which defines the arrival and service time distributions. Here, arrivals assume Poisson process with parameter λ, service times are iid variables following an arbitrary distribution and one service channel (traffic light) exists.

3.4 Compressed $M_{+\Delta}/G_{+\Delta}/1$ Queueing System

The service times in the stochastic delay component can be analyzed effectively using the compressed $M_{+\Delta}/G_{+\Delta}/1$ queueing system because of its distribution. The system is drawn from compressed queueing processes theory so as to estimate statistical measures of traffic delay in case of large variations of service times. In this model, $M_{+\Delta}$ represents the exponential shifted distribution for the inter arrival times, $G_{+\Delta}$ represents the general shifted distribution of service times and 1 implies a single service channel (traffic light). The level of service in this model is basically described by the mean and variance of the service time spent by a vehicle in the queue. The compressed queueing processes used in this study are based on two assumptions:

(i). The service time rate for a compressed model, denoted by μ' and given by

$$\mu' = \frac{\mu}{1-\mu\Delta}. \tag{3.1}$$

(ii). The arrival rate for the compressed model, denoted by λ' and given by

$$\lambda' = \frac{\lambda}{1-\lambda\Delta}. \tag{3.2}$$

3.5 Statistical Software

In this study, we employed the use of MATLAB (matrix laboratory) software for simulation as evident in chapter 5. MATLAB is a numerical computing environment and fourth-generation programming language developed by MathWorks. In the next chapter, we describe the development of overall traffic delay model.

3.6 Sampling Method

The traffic data was collected from a randomly selected three days of the week in the month of February, 2013, that is, on 20th, 21st and 22nd February, 2013 from 5:13 PM to 6:10 PM daily at Kenyatta Avenue-Kimathi Street signalized intersection. These data represented the traffic data on general weekdays. The traffic data (in seconds) collected at the intersection were: G, Y, AR, R, l_1, l_2, t and Δ representing green time, amber (yellow) time, all red time, red time, start-up lost time, clearance lost time, evaluation period and minimal time distance between vehicles respectively. The traffic periods and distance between vehicles were measured by a clock-timer and tape-measure respectively.

CHAPTER FOUR

OVERALL TRAFFIC DELAY MODEL

4.1 Introduction

This chapter presents the development of a model for estimating the mean and variance of overall traffic delay at a signalized intersection. Section 4.2 discusses the formulation of the problem and assumptions of the queueing models. Section 4.3 provides the mean and variance of a deterministic delay component while that of stochastic delay component are provided in Section 4.4. Finally, Section 4.5 discusses the moments of overall traffic delay.

4.2 Problem Formulation

Consider a cumulative arrival and departure of vehicles in a signalized intersection for the time interval $0,T$. The time taken by a vehicle in the queue herein referred to as overall traffic delay is denoted by D_t. Here, D_t comprises of deterministic and stochastic delay components and can be broken as follows:

$$D_t = D_{t_1} + D_{t_2}, \tag{4.1}$$

where D_{t_1} is the deterministic delay component representing a delay that is incurred by a vehicle with uniform arrival times and departures within the time interval $\left[t, t+c_y\right]$ while D_{t_2} is the stochastic delay component representing the delay that is caused by random queues resulting from the random nature of arrivals. The idea here is to solve the stochastic Equation (4.1). And, before we solve it, we make the following assumptions:

a) The intersection consists of only a single lane controlled by a fixed-time signal and unlimited space for queueing;

b) The vehicles' arrival at the intersection is either uniform or random variable following a Poisson process and no initial queue is present at the time when a prediction is performed;

c) The vehicle time prediction horizon is assumed to be equal to the signal cycle time.

4.3 Deterministic Delay Component

Deterministic delay component as described in (4.1) is denoted by D_{t_1}. In this section, we shall be interested in the computation of the mean and variance of D_{t_1}. The mean and variance of the deterministic delay component is estimated by deterministic queueing model D/D/1, where the first D represents uniform arrivals with parameter λ, the second D representing constant departures with parameter μ and 1 representing one service channel (traffic light) existing. In Figure 4.1 below, we present a diagrammatic description of deterministic delay process.

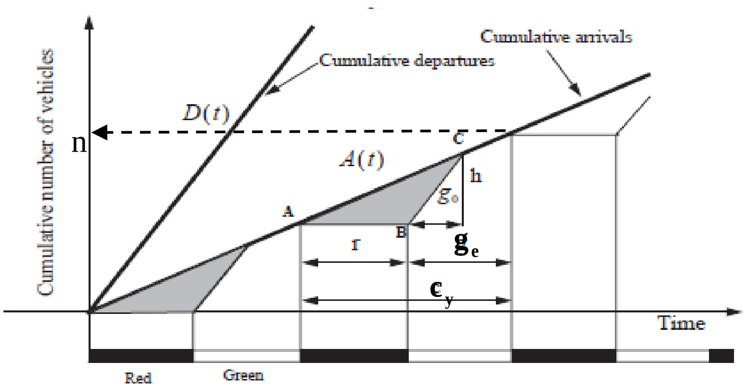

Figure 4.1: Deterministic component of overall traffic delay.

The Figure displays the deterministic delay component of overall traffic delay at a signalized intersection. From the Figure, *D(t)* and *A(t)* represents the cumulative departures and arrivals, respectively. The area under cross-sectional area covered by triangle *ABC* represents the total deterministic delay at the intersection. From the figure (4.1), we can determine the statistical measures: mean and variance.

4.3.1 The Mean

To compute the mean, we assume that vehicle arrivals and departures are uniformly distributed with rates λ and μ, respectively. The mean delay to vehicles for this case can then be easily determined from the figure shown in Figure (4.1). The figure shows a typical cumulative arrival/departure graph against time for uniform arrival rate approach to an intersection. The slope of the cumulative arrival line is the uniform arrival rate in vehicles per unit time, denoted by λ. The slope of the cumulative departure line is sometimes zero (when the light is red) and sometimes ρ (when the light is green); where ρ is the traffic intensity obtained as $\rho=\lambda/\mu$.

Upon utilizing D/D/1 queueing system and the theory behind it, we compute the mean. Notice that the duration of c_y at the signalized intersection is given by

$$c_y = r + g_e, \tag{4.2}$$

where c_y, r and g_e are provided in the list of symbols. From Figure (4.1), we note that g_o denotes the time necessary for the queue to dissipate. Here, the queue must dissipate before the end of g_e. But if the queue doesn't dissipate before the end of g_e, the queue would escalate indefinitely. From this statement, we deduce that

$$g_o \leq g_e. \tag{4.3}$$

Condition (4.3) is satisfied if the total number of vehicle arrivals during c_y is less than or equal to the total number of vehicle departures during g_e. That is,

$$\frac{\lambda}{\mu} \le \frac{g_e}{c_y}. \tag{4.4}$$

Derivation of (4.4) is provided in Appendix A. Also from Figure (4.1), we can deduce that vehicles arrive during time period $r + g_0$ and depart during the time period $\frac{c_y}{g_e}.g_o$.

Since the total number of vehicle arrivals equals the total number of vehicle departures, we have that

$$\lambda \;\; r + g_o \;\; = \mu \frac{c_y}{g_e}.g_o$$

$$\left(\mu \frac{c_y}{g_e} - \lambda \right) g_o = \lambda r. \tag{4.5}$$

The time period g_o required for queue to dissipate is

$$g_o = \frac{\lambda r}{\left(\mu \frac{c_y}{g_e} - \lambda \right)}. \tag{4.6}$$

Equation (4.6) simplifies to

$$g_o = \frac{\lambda r / \mu}{\left(\frac{c_y}{g_e} - \frac{\lambda}{\mu} \right)}. \tag{4.7}$$

Upon writing (4.7) in terms of traffic intensity defined by $\rho=\lambda/\mu$, we have

$$g_o = \frac{\rho.r}{\left(\frac{c_y}{g_e} - \rho \right)}. \tag{4.8}$$

From the figure, it can be seen that D_{t_1} is given by

$$D_{t_1} = \sum_{i=1}^{n} d \;\; i \;\; ,$$

where *d(i)* is the shaded cross-sectional area in Figure (4.1). Assuming that *n* is large enough so that the discrete sum of *d(i)* is equal to the area of the cross-sectional area covered by triangle *ABC* in the figure the following can be written:

$$D_{t_1} = \frac{1}{2} h \;\; c_y - g_e \quad .$$

And here, h can be easily determined by noting that

$$h = \lambda \;\; r + g_o \quad .$$

Therefore,

$$D_{t_1} = \frac{\lambda r}{2} \;\; r + g_o \quad . \tag{4.9}$$

Upon utilizing (4.8), we have

$$D_{t_1} = \frac{\lambda r^2}{2\left(1 - \frac{g_e}{c_y}\rho\right)} . \tag{4.10}$$

To obtain the expected deterministic delay, we divide D_{t_1} by the total number of vehicles in a cycle, that is, λc_y to give

$$E\left[D_{t_1}\right] = \frac{c_y\left(1 - \frac{g_e}{c_y}\right)^2}{2\left(1 - \frac{g_e}{c_y}\rho\right)} \tag{4.11}$$

as the mean of the deterministic component, D_{t_1} . Next, we compute the variance of D_{t_1} .

4.3.2 The Variance

The conventional way of computing the $Var\left[D_{t_1}\right]$ is

$$Var\left[D_{t_1}\right] = E\left[D_{t_1}^{\;2}\right] + \;\; E\left[D_{t_1}\right]^{\;2} . \tag{4.12}$$

Since (4.11) provides us with $E\left[D_{t_1}\right]$, we compute for $E\left[D_{t_1}{}^2\right]$. To begin with, we compute $D_{t_1}{}^2$. Again, we assume n large enough so that the discrete sum of $d(i)^2$ is equal to the volume of the cross-sectional area covered by triangle ABC in the figure, that is

$$D_{t_1}{}^2 = \sum_{i=1}^{n} d\left(i\right)^2$$

$$D_{t_1}{}^2 = \frac{1}{3} h\left(c_y - g_e\right)^2.$$

Upon substituting for h, we get

$$D_{t_1}{}^2 = \frac{\lambda r^2}{3}\left(r + g_o\right). \tag{4.13}$$

By (4.2) and (4.8), (4.13) simplifies to

$$D_{t_1}{}^2 = \frac{\lambda c_y{}^3\left(1 - \frac{g_e}{c_y}\right)}{3\left(1 - \frac{g_e}{c_y}\rho\right)}.$$

To obtain $E\left[D_{t_1}{}^2\right]$ we divide the above result by the total number of vehicles, λc_y

$$E\left[D_{t_1}{}^2\right] = \frac{c_y{}^2\left(1 - \frac{g_e}{c_y}\right)^3}{3\left(1 - \frac{g_e}{c_y}\rho\right)}. \tag{4.14}$$

Equation (4.14) is the second moment of the deterministic delay component. Thus, utilizing (4.11) and (4.14), we have

$$Var\left[D_{t_1}\right] = \frac{c_y{}^2\left(1 - \frac{g_e}{c_y}\right)^3\left(1 + 3\frac{g_e}{c_y} - 4\frac{g_e}{c_y}\rho\right)}{12\left(1 - \frac{g_e}{c_y}\rho\right)^2}, \tag{4.15}$$

as the variance of the deterministic component, D_{t_1}. Similarly, we compute the mean and variance of the stochastic component, D_{t_2}.

4.4 Stochastic Delay Component

In this section, we computed the statistical measures of the stochastic delay component, that is, mean and variance of D_{t_2}. The component is established through a coordinate transformation technique based on the queueing system $M_{+\Delta}/G_{+\Delta}/1$ with the usage of compressed queueing processes. Under this system, the vehicles arrive at the intersection in a Poisson process. The inter-arrival times follow a shifted exponential distribution given by

$$A(t) = 1 - \lambda e^{-\lambda t}. \tag{4.16}$$

The service times are iid random variables following a general distribution characterized by its Probability density function determined by $f_X(x)$ or $F_X(x)$. Suppose N_t vehicles are on the queue at time t and, R_t being the residual service time of vehicle j. Residual service time herein, is the time until the vehicle found by vehicle j being served by the traffic light completes the service. Then for us to describe the state of the queueing system at time t, we need to compute the value of N_t, the probability that j vehicles are on the queue by

$$P_r(N_t = j) = \pi_j. \tag{4.17}$$

We shall use the generating function technique to compute (4.17) as follows

$$\mathrm{P}(s) = \sum_{j\geq 0} s^j P_r(N_t = j), \tag{4.18}$$

where $\mathrm{P}(s)$ is the transform of the system size distribution. Equation (4.18) simplifies to

$$P(s) = \sum_{j\geq 0} s^j \pi_j,$$

with $\pi_j = \sum_{i \geq 0} \mathrm{P}_{ij}\pi_i$ for $j \geq 0$, where P_{ij} is a transition probability, (4.18) further becomes

$$\mathrm{P}(s) = \sum_{i \geq 0}\sum_{j \geq 0} \pi_i P_{ij} s^j \quad . \tag{4.19}$$

Note that the transition $0 \to j$ occurs if and only if j arrivals occur in the service time following an idle period, whereas the transition $i \to j$ (with *i > 0*) occurs if and only if $j - i + 1$ arrivals occur during a service time. If q_j is the probability of j arrivals in a service time and $Q(s)$ is the generating function of q_j, we have

$$P(s) = \pi_0 \frac{Q(s)(1-s)}{Q(s) - s} . \tag{4.20}$$

The derivation of (4.20) is provided in Appendix B. The matrix of (4.20) takes the form

$$\mathrm{P}(s) = \begin{bmatrix} q_0 & q_1 & q_2 & \cdots \\ 0 & q_0 & q_1 & \cdots \\ 0 & 0 & q_0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} . \tag{4.21}$$

To compute q_j, first note that N_t follow a Poisson distribution with parameter λt at time t. Thus,

$$q_j = \int_{t=0}^{\infty} \frac{(\lambda t)^j}{j!} e^{-\lambda t} f_X(t)\, dt , \tag{4.22}$$

where $f_X(t)$ is the service time distribution. The generating function of q_j herein denoted by $Q(s)$ is given by

$$Q(s) = \sum_{i \geq 0} q_j s^j . \tag{4.23}$$

And by using (4.22), Equation (4.23) simplifies to

$$Q(s) = \int_{t=0}^{\infty} \sum_{j=0}^{\infty} e^{-\lambda t(1-s)} f_X(t)\, dt . \tag{4.24}$$

Notice that in (4.24), the Laplace transform of the service time distribution is

$$Q(s) = X^{*}\left(\lambda(1-s)\right), \tag{4.25}$$

by definition, X^{*} in (4.25) is referred to as the service time transform. Equation (4.25) is also referred to as the Laplace-Stieltjes transform (LST) or Pollaczek-Khintchine (P-K) transform of the service time distribution with first and second moments denoted by $E[X]$ and $E\left[X^{2}\right]$, respectively.

Next, we compute π_0 in (4.20) by employing L'Hospital's rule with the assumption that $P(1) = Q(1) = 1$

$$P'(s) = \frac{d}{ds} \cdot$$

$$P'(s) = \pi_0 \left(\frac{Q'(s)(1-s) - Q(s)}{Q'(s) - 1} \right).$$

Upon taking the limit $s \to 1$, we get

$$\lim_{s \to 1} P'(s) = \lim_{s \to 1} \cdot$$

$$\lim_{s \to 1} P'(s) = \frac{-\pi_0}{Q'(1) - 1}. \tag{4.26}$$

Applying Little's theorem, defined by $\rho = \lambda E[X]$ to (4.26), we have

$$\pi_0 = 1 - \rho. \tag{4.27}$$

Thus, (4.20) can be written as

$$\mathrm{P}(s) = \frac{(1-\rho)\, X^{*}\left(\lambda(1-s)\right)(1-s)}{X^{*}\left(\lambda(1-s)\right) - s}. \tag{4.28}$$

Equation (4.28) will be vital in the derivation of $Var\left[D_{t_2}\right]$. Next is the computation of the mean of D_{t_2}.

4.4.1 The Mean

First, we break D_{t_2} of (4.1) into W_t and X_t, where W_t is the waiting time for vehicle j and X_t is the service time for vehicle j. Therefore, D_{t_2} is given by

$$D_{t_2} = W_t + X_t .$$

Thus expectation of D_{t_2} is

$$E\left[D_{t_2}\right] = E\left(W_t\right) + E\left(X_t\right). \tag{4.29}$$

To obtain $E\left[D_{t_2}\right]$, first we compute $E\left(W_t\right)$. Assuming First Come First Served (FCFS) discipline, we have

$$W_t = \rho R_t + X_{t-1} + X_{t-2} + \ldots.. + X_{t-Q_t}$$

$$W_t = \rho R_t + \sum_{i=1}^{Q_t} X_{t-i} \tag{4.30}$$

where W_t, ρ, R_t, X_t and Q_t are as provided in the list of symbols. Therefore $E\left(W_t\right)$ is

$$E\left(W_t\right) = \rho E\left(R_t\right) + E\left[\sum_{i=1}^{Q_t} X_{t-i}\right].$$

The Q_t as defined in the list of symbols is a random variable hence,

$$E\left(W_t\right) = \rho E\left(R_t\right) + EE\left[X_t \middle| Q_t\right].$$

Since X_t is independent of Q_t, we have

$$E\left(W_t\right) = \rho E\left(R_t\right) + E\left(X_t\right) \cdot E\left(Q_t\right).$$

Upon taking the limit $t \to \infty$, we get

$$\underset{t\to\infty}{Limit}\, E\left(W_t\right) = \underset{t\to\infty}{Limit}\left(\rho E\left(R_t\right) + E\left(X_t\right) \cdot E\left(Q_t\right)\right).$$

Hence,

$$E\left(W\right) = \rho E\left(R\right) + E\left(X\right) E\left(Q\right). \tag{4.31}$$

The expectations $E[R]$ and $E[Q]$ in (4.31) are those observed by arriving vehicle at the intersection. From Poisson Arrivals See Time Averages (PASTA) property, the statistical measures (mean, variance and distribution) of the number of vehicles in the queueing system observed by an arrival is the same as those observed by an independent Poisson inspector. If we assume that vehicles arrive at the intersection in a Poisson process, then the expected number of vehicles in the queue excluding the one being served is given by

$$E[Q] = \lambda E[W],$$

utilizing this relation in (4.31), we get

$$E[W] = \frac{\rho E[R]}{1-\rho}, \tag{4.32}$$

where, ρ is the traffic intensity defined by $\rho=\lambda E[X]$ (Little's law).

To compute $E[R]$ in (4.32), consider Figure (4.2) below.

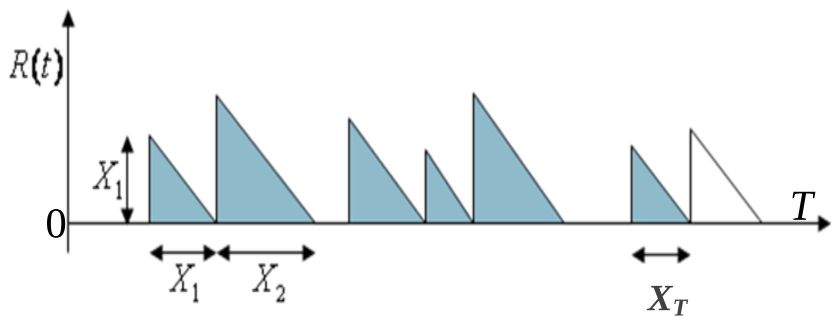

Figure 4.2: Diagram representing long-term residual service time.

In Figure 4.2, we present a diagrammatic description of a long-term expected residual time.

To compute the (unconditional) mean residual service time $E[R]$, consider the process $\{R(t), t \geq 0\}$ where $R(t)$ is the residual service time of the vehicle in service at time t. And consider a very long time interval $[0,T]$. Then

$$E(R)=\frac{1}{T}\int_0^T R(t)\,d(t). \tag{4.33}$$

Let $X(T)$ be the number of service completions by time T and X_i the i^{th} service time. Notice that the function $R(t)$ takes the value zero when there is no vehicle in service and jumps to the value of X_i at the time the i^{th} service time commences. During a service time it linearly decreases with rate of one and reaches zero at the end of a service time. Therefore, $E\left[R(t)\right]$ is equal to the sum of the areas of $X(T)$ isosceles right triangles where the side of the i^{th} triangle is X_i. For large T, we can ignore the last possibly incomplete triangle to obtain

$$E(R)=\frac{1}{T}\sum_{i=1}^{X(T)}\frac{1}{2}X_i^2$$

$$E(R)=\frac{1}{2}\cdot\frac{X(T)}{T}\cdot\frac{1}{X(T)}\cdot\sum_{i=1}^{X(T)}X_i^2 .$$

Letting T approach infinity and employing the law of large numbers, the latter gives

$$E(R)=\frac{1}{2}\lambda E\left[X^2\right], \tag{4.34}$$

where $E\left[X^2\right]$ is the second moment of the service time.

By (4.32) and (4.34), we obtain

$$E(W)=\frac{\rho}{2(1-\rho)}\lambda E\left[X^2\right]. \tag{4.35}$$

And utilizing (4.35) in (4.29) we establish the expected time a vehicle spends in the queue, $E\left[D_{t_2}\right]$ as

$$E\left[D_{t_2}\right]=\frac{\rho}{2(1-\rho)}\lambda E\left[X^2\right]+E(X). \tag{4.36}$$

Upon employing the compressed queueing processes as described in (3.5) and (3.6), Equation (4.36) reduces to

$$E\left[D_{t_2}\right]=\frac{\rho\lambda E\left[X^2\right]}{2\left(1-\rho\right)\left(1-\lambda\Delta\right)}+E\left(X\right). \qquad (4.37)$$

Finally, we compute the variance of D_{t_2}.

4.4.2 The Variance

In this section, we are interested in the computation of $Var\left[D_{t_2}\right]$. First note that

$$D_{t_2}=W_t+X_t.$$

Therefore,

$$Var\left[D_{t_2}\right]=Var\left(W_t\right)+Var\left(X_t\right)-2Cov\left(W_t\cdot X_t\right),$$

but we know that W_t and X_t are independent random variables, thus

$$Var\left[D_{t_2}\right]=Var\left(W_t\right)+Var\left(X_t\right). \qquad (4.38)$$

Equation (4.38) is vital in the derivation in this section. Note that D_{t_2} is a sum of two independent random variables, that is, W_t and X_t. If the generating function of X_t is $X_t^*\left(s\right)$ and that of W_t is $W_t^*\left(s\right)$, the joint transformed probability generating function of D_{t_2} is

$$P_t^*\left(s\right)=W_t^*\left(s\right)+X_t^*\left(s\right),$$

where $P^*\left(s\right)$ and $X^*\left(s\right)$ are P-K transforms of the queueing system size and service time distributions respectively. Taking limits as $t\to\infty$, we have

$$\lim_{t\to\infty}P_t^*\left(s\right)=\lim_{t\to\infty}\left(W_t^*\left(s\right)+X_t^*\left(s\right)\right).$$

Thus,

$$P^*(s) = W^*(s) + X^*(s). \tag{4.39}$$

Since the transform of the sum of two independent random variables is equivalent to the product of their transforms for instance see Ivo and Jacques, 2002, Section 2.3, then

$$P^*(s) = W^*(s) \times X^*(s).$$

Upon utilizing (4.25) and (4.28), $W^*(s)$ is given by

$$W^*(s) = \frac{(1-\rho)s}{\lambda X^*(s) + s - \lambda}. \tag{4.40}$$

By Little's law, $\rho = \lambda E(X)$, Equation (4.40) simplifies to

$$W^*(s) = \frac{1-\rho}{1-\rho R^*(s)}. \tag{4.41}$$

Derivation of (4.41) is given in Appendix C. Equation (4.41) is the P-K transform of waiting time distribution, hence to get the first and second moments of waiting time, we differentiate with respect to s and set $s=0$ to get

$$W'^*(s) = \frac{d}{ds}(\cdot)\Big|_{s=0}.$$

Hence,

$$E(W) = \frac{\rho E(R)}{1-\rho}, \tag{4.42}$$

a result similar to (4.32). Again, differentiating (4.41) twice with respect to s and set $s=0$, we get

$$W''^*(s) = \frac{d^2}{ds^2}(\cdot)\Big|_{s=0}.$$

Hence,

$$E\left[W^2\right] = 2\left(E(W)\right)^2 + \frac{\rho E\left[R^2\right]}{1-\rho}. \tag{4.43}$$

To compute $E\left[R^2\right]$ again, we consider Figure (4.2) and deduce that

$$E\left[R^2\right]=\frac{1}{T^2}\int_0^T R\ \ t\ \ \ dt\,,$$

which simplifies to

$$E\left[R^2\right]=\frac{\lambda E\left[X^3\right]}{3}. \tag{4.44}$$

Utilizing (4.34) and (4.44) in Equations (4.42) and (4.43), respectively, we get

$$E\ \ W\ \ =\frac{\rho\lambda E\left[X^2\right]}{2\ \ 1-\rho} \tag{4.45}$$

and

$$E\left[W^2\right]=\rho\lambda\left(\rho\lambda\left(\frac{E\left[X^2\right]}{1-\rho}\right)^2+\frac{E\left[X^3\right]}{3\ \ 1-\rho}\right). \tag{4.46}$$

Then, to compute $Var\left[D_{t_2}\right]$, we use (4.29) and (4.46) to obtain $E\left[D_{t_2}{}^2\right]$ and $E\left[D_{t_2}\right]^{\ 2}$ as

$$E\left[D_{t_2}{}^2\right]=\rho\lambda\left(\rho\lambda\left(\frac{E\left[X^2\right]}{1-\rho}\right)^2+\frac{E\left[X^3\right]}{3\ \ 1-\rho}\right)+E\left[X^2\right] \tag{4.47}$$

and

$$E\left[D_{t_2}\right]^{\ 2}=\left(\frac{\rho\lambda E\left[X^2\right]}{2\ \ 1-\rho}+E\ \ X\ \ \right)^2, \tag{4.48}$$

respectively. Having obtained (4.47) and (4.48), Equation (4.39) simplifies to

$$Var\left[D_{t_2}\right]=\frac{3}{4}\left(\frac{\rho\lambda E\left[X^2\right]}{1-\rho}\right)^2+\frac{\rho\lambda E\left[X^3\right]}{3\ \ 1-\rho}+\left(1-\frac{\rho\lambda E\ \ X}{1-\rho}\right)E\left[X^2\right]-\ \ E\ \ X\ \ ^{2}. \tag{4.49}$$

Employing the use of compressed queueing processes, (4.49) becomes

$$Var\left[D_{t_2}\right]=\frac{3}{4}\left(\frac{\rho\lambda E\left[X^2\right]}{(1-\rho)\cdot(1-\lambda\Delta)}\right)^2+\frac{\rho\lambda E\left[X^3\right]}{3(1-\rho)\cdot(1-\lambda\Delta)}+\left(1-\frac{\rho\lambda E(X)}{(1-\rho)\cdot(1-\lambda\Delta)}\right)E\left[X^2\right]-(E(X))^2. \tag{4.50}$$

The next section considers the combined D_{t_1} and D_{t_2}.

4.5 The Moments of Overall Traffic Delay

Notice from (4.1) that D_t can be split into two independent components, that is, D_{t_1} and D_{t_2}. In the previous sections, we have confined ourselves in the computation of mean and variance of D_{t_1} and D_{t_2}. In this section, we amalgamate the two sections to obtain $E(D_t)$ and $Var(D_t)$. To obtain $E(D_t)$, we have

$$E(D_t)=E\left[D_{t_1}\right]+E\left[D_{t_2}\right]. \tag{4.51}$$

Utilizing (4.11) and (4.37), Equation (4.51) becomes

$$E(D_t)=\frac{c_y\left(1-\frac{g_e}{c_y}\right)^2}{2\left(1-\frac{g_e}{c_y}\rho\right)}+\frac{\rho\lambda E\left[X^2\right]}{2(1-\rho)(1-\lambda\Delta)}+E(X). \tag{4.52}$$

Similarly, $Var(D_t)$ is given by

$$Var(D_t)=Var\left[D_{t_1}\right]+Var\left[D_{t_2}\right]-2Cov(D_{t_1}.D_{t_2}), \tag{4.53}$$

and since D_{t_1} and D_{t_2} are independent components, we utilize (4.15) and (4.50) in (4.53) to give $Var(D_t)$ as

$$Var\left(D_t\right) = \frac{c_y\left(1-\frac{g_e}{c_y}\right)^3\left(1+3\frac{g_e}{c_y}-4\frac{g_e}{c_y}\rho\right)}{12\left(1-\frac{g_e}{c_y}\rho\right)^2} + \frac{3}{4}\left(\frac{\rho\lambda E\left[X^2\right]}{\left(1-\rho\right)\cdot\left(1-\lambda\Delta\right)}\right)^2 + \frac{\rho\lambda E\left[X^3\right]}{3\left(1-\rho\right)\cdot\left(1-\lambda\Delta\right)}$$

$$+\left(1-\frac{\rho\lambda E\left[X\right]}{\left(1-\rho\right)\cdot\left(1-\lambda\Delta\right)}\right)E\left[X^2\right]-\left(E\left[X\right]\right)^2. \quad (4.54)$$

In the next chapter, we apply our developed model on the real traffic data.

CHAPTER FIVE

RESULTS AND DISCUSSION

5.1 Introduction

In this chapter, we apply the developed overall traffic delay model on real traffic data collected at Kenyatta Avenue-Kimathi Street signalized intersection between 20th and 22nd February, 2013. The intermediate results from the data are given and simulation on the developed models using MATLAB software is performed for traffic intensities ranging from 0.1 to 1.9. This chapter is summarized as follows: Section 5.2 provides computation of parameters necessary for simulating the developed models. The simulation of $E(D_t)$ and $Var(D_t)$ are provided in Sections 5.3 and 5.4 of this chapter, respectively. The application of $Var(D_t)$, that is, the variability of level of service (LOS) is provided in Section 5.5.

5.2 Computation of Parameters

For the simplicity of sampling and measurement, we assumed that the data collected on 20th, 21st and 22nd February, 2013 from 5:13 PM to 6:10 PM daily represented the traffic data on general weekdays. The traffic measurements (in seconds) recorded were: G, Y, AR, R, l_1, l_2, t and Δ representing green time, amber (yellow) time, all red time, red time, start-up lost time, clearance lost time, evaluation period and minimal time distance between vehicles, respectively. Figure (5.1) below describes a typical sequence of lights at the signalized intersection.

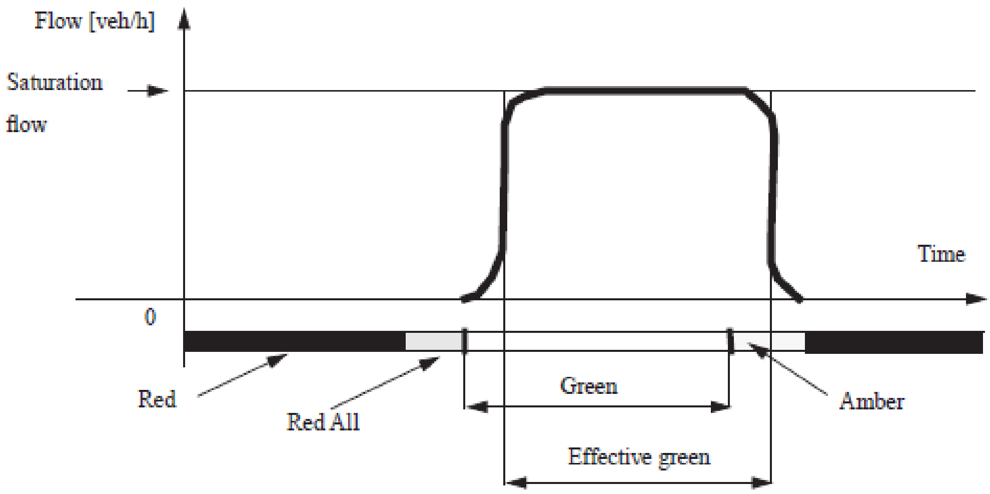

Figure 5.1: Diagram showing a typical sequence of lights at the signalized intersection.

Considering a single lane controlled by a fixed-time traffic signal, we recorded the duration of the green lights that allow the vehicles to go through Kenyatta Avenue-Kimathi Street intersection and the number of vehicles passing during the effective green lights after every cycle time of 180 seconds. The data collected is provided in Tables 5.1 - 5.3 (Appendix D).

To compute Δ, we also recorded the speed of vehicles on the queue and the distance between them on Friday, 22nd February, 2013, from 5:13 PM – 6:10 PM. The data is given in Table 5.4 (see Appendix D). Assuming this data to be a representative for all weekdays, we compute the average weekday speed of vehicles in the queue and distance in between them. From the data on Table 5.1 – 5.3, the average effective green time is

$$g_e = \frac{1}{3} \times \left(\frac{1396}{20} + \frac{1356}{20} + \frac{1342}{20} \right)$$

$$= 68.23 \text{ sec.} \tag{5.1}$$

Average service time during the green light is

$$\overline{x_{g_e}} = \frac{1}{3} \times \left(\frac{1396}{1405} + \frac{1356}{1383} + \frac{1342}{1302} \right)$$

$$= 1.002 \text{ sec.} \tag{5.2}$$

Average effective red time is

$$\overline{x_{g_e}} = \frac{1}{3} \times \left(\frac{2204}{20} + \frac{2240}{20} + \frac{2256}{20} \right)$$

$$= 111.67 \text{ sec.} \tag{5.3}$$

In our model, we assume that the traffic light is always running. Thus, service time of the first vehicle passing through the intersection when a green light turns on is considered to be equal to the red light duration. We denote the average service time for that vehicle as $\overline{x_r}$ given by

$$\overline{x_r} = t_r = 111.67 \text{ sec.}$$

For each green light during 5:13 PM – 6:10 PM, there is only one vehicle which has the service time $\overline{x_r}$. All the other vehicles have the service time $\overline{x_{g_e}}$. The green lights turn on 20 times, so the number of vehicles with service time $\overline{x_r}$ is equal to 20. The probability that a vehicle has a service time $\overline{x_r}$ is given by

$$\Pr\left[X = \overline{x_r}\right] = \frac{1}{3} \times \left(\frac{20}{1405} + \frac{20}{1383} + \frac{20}{1302} \right)$$

$$= 0.015. \tag{5.4}$$

And the probability that a vehicle has service time $\overline{x_{g_e}}$ is

$$\Pr\left[X = \overline{x_{g_e}}\right] = 1 - \Pr\left[X = \overline{x_r}\right]$$

$$= 0.985. \tag{5.5}$$

Thus, the average service time becomes

$$\overline{x} = \Pr\left[X = \overline{x_{g_e}}\right].\overline{x_{g_e}} + \Pr\left[X = \overline{x_r}\right].\overline{x_r}$$

$$= 2.66 \text{ sec.}$$

The average service rate is

$$\mu = \frac{1}{2.66} = 0.38 \text{ sec.} \tag{5.6}$$

Based on the data (Table 5.4), we can get the average speed of a vehicle in the queue during the time period (5:13 PM - 6:10 PM) as

$$\frac{256}{20} = 12.8 \text{ Km/h.}$$

Converting the above result to M/s, we have

$$12.8 \times \frac{1000}{3600} = 3.56 \text{ M/s.} \tag{5.7}$$

The average distance between the vehicles in the queue is

$$\bar{d} = \frac{25.7}{20} = 1.285 \text{ M.} \tag{5.8}$$

Using (5.8), Δ is obtained as

$$\Delta = \frac{1.285}{3.56}$$

$$= 0.36 \text{ sec.} \tag{5.9}$$

5.3 Simulation of $\mathbf{E}\ \mathbf{D_t}$

Using Equation (4.52) and the collected data, we obtained Figure 5.2 by MATLAB software when we assumed that service times follow Exponential distribution with parameter $1/\lambda$.

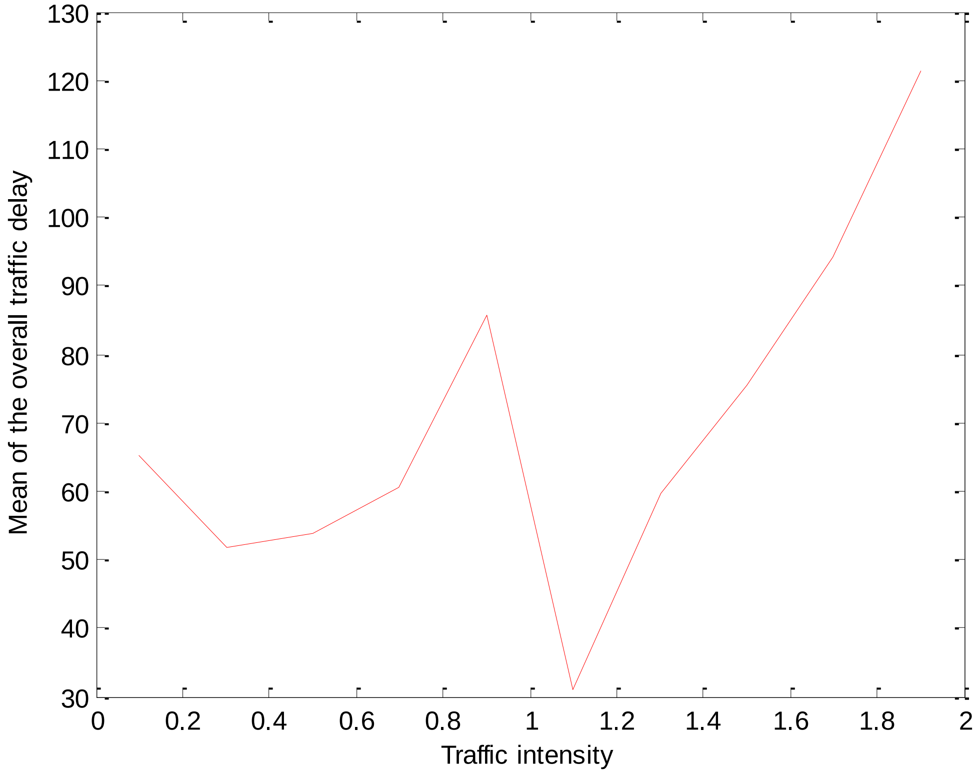

Figure 5.2: Diagram describing relationship between $\mathbf{E\left[D_t\right]}$ and ρ.

To be able to explain this figure, we split $E\left[D_t\right]$ into $E\left[D_{t_1}\right]$ and $E\left[D_{t_2}\right]$ that yields Figure 5.3 below.

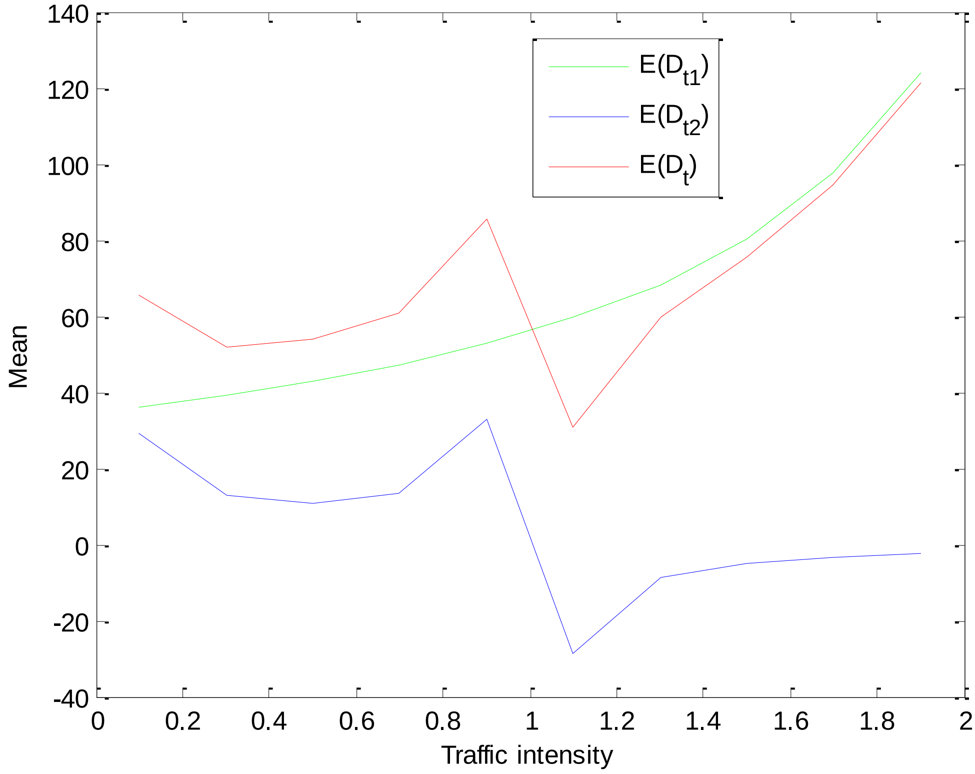

Figure 5.3: $E\left[D_{t_1}\right], E\left[D_{t_2}\right]$ and $E\ D_t$ versus ρ using Exponential distribution of service times.

From Figure 5.3, it is clear to note that the stochastic delay model is only applicable to undersaturated conditions ($\rho < 1$) and estimate infinite delay when arrival flow approaches capacity. However, when arrival flow exceeds capacity oversaturated queues exist and continuous delay occurs. It is also evident that the deterministic delay model estimates continuous delay, but it does not completely deals with the effect of randomness when the arrival flows are close to capacity, and also fail when the traffic intensity is between 1.0 and 1.1. The figure shows that both components of our overall traffic delay model are incompatible when the traffic intensity is equal to 1.0. Therefore, our overall traffic delay model is used to fill the gap between the two models and also give more realistic results in the estimation of delay at signalized intersections. It predicts the delay for both undersaturated and oversaturated traffic conditions without having any discontinuity at the traffic intensity of 1.0. Harmonizing $E\left[D_{t_1}\right]$ and $E\left[D_{t_2}\right]$ components result into $E\left[D_t\right]$ described in Figure 5.2 above.

Similarly, with the assumption of service times following Gamma distribution, we obtained Figure 5.4 below by MATLAB.

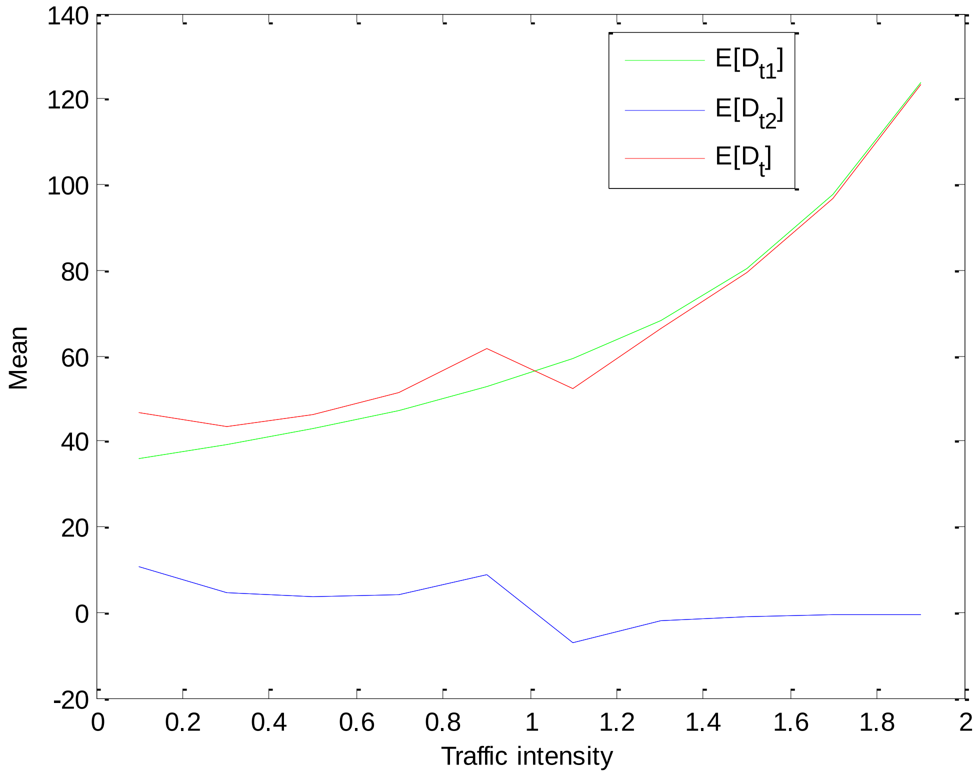

Figure 5.4: $\mathbf{E\left[D_{t_1}\right], E\left[D_{t_2}\right]}$ and $\mathbf{E\left[D_t\right]}$ versus $\boldsymbol{\rho}$ using Gamma distribution of service times.

We depict that under this assumption, $E\left[D_t\right]$ increases rapidly with ρ than in the Exponential assumption under oversaturated traffic conditions ($\rho \geq 1.15$), although the general behaviour is similar to the Exponential assumption. From the figure, $E\left[D_{t_1}\right]$ remains the same as that of exponential distribution of service times. Comparing Figure

5.3 and Figure 5.4, Figure 5.3 estimates a lower value of $E(D_t)$ than Figure 5.4, that is, Figure 5.4 estimates $E(D_t)$ to be 43.12 seconds while Figure 5.3 estimates $E(D_t)$ to be 30.93 seconds. Also, Figure 5.4 estimates higher values of $E(D_t)$ as $\rho \geq 1.5$. This is contrary to what $E(D_t)$ with exponential distribution of service times estimates. Therefore, exponential distribution of service times is far much preferred since we are interested in a reduced mean of overall traffic delay at the intersection.

5.4 Simulation of $\mathbf{Var(D_t)}$

Simulating $Var(D_t)$ by Equation (4.55), we obtained the harmonized variance as shown in Figure 5.5 below.

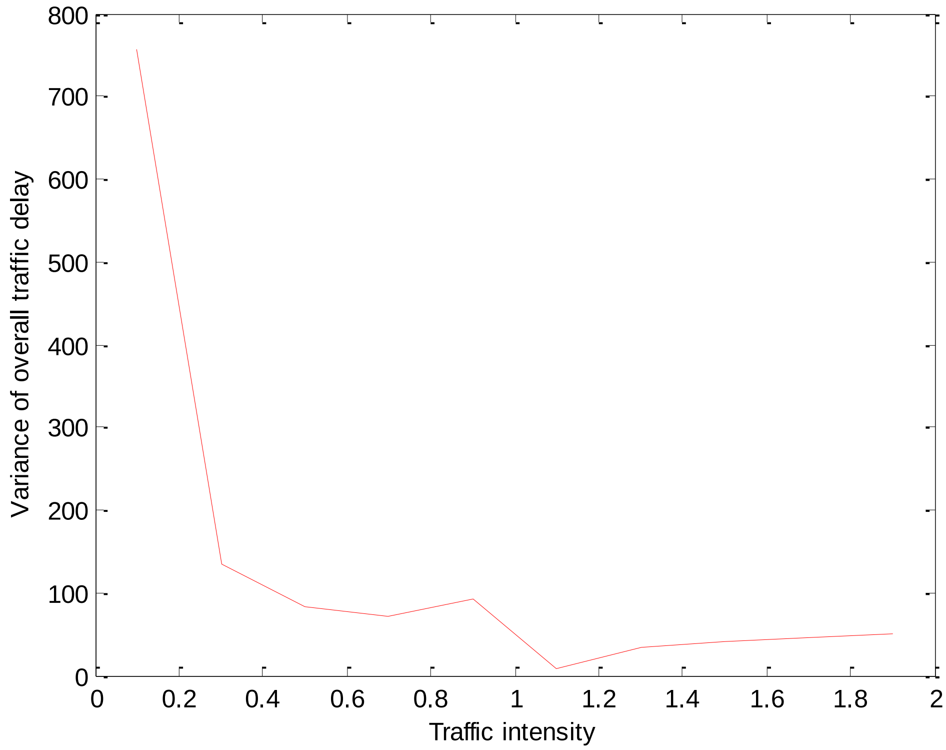

Figure 5.5: $\mathbf{Var(D_t)}$ versus $\boldsymbol{\rho}$ using Exponential distribution of service times.

When the traffic intensity is between 0 and 0.3, the traffic flows are independent of each other since the service rate of the traffic light is higher than the vehicle arrival rate. This results in a rapid decrease of $Var\ D_t$. But as ρ approaches 1.0, the vehicle transition begin to depend on the traffic flows resulting to a slower decrease in $Var\ D_t$. However, as ρ goes beyond 1.0, $Var\ D_t$ slowly increases. To investigate the major contributor to $Var\ D_t$ by D_{t_1} and D_{t_2}, we plot the graphs of $Var\ D_t$, $Var\left[D_{t_1}\right]$ and $Var\left[D_{t_2}\right]$ versus ρ as shown in Figure 5.6 below.

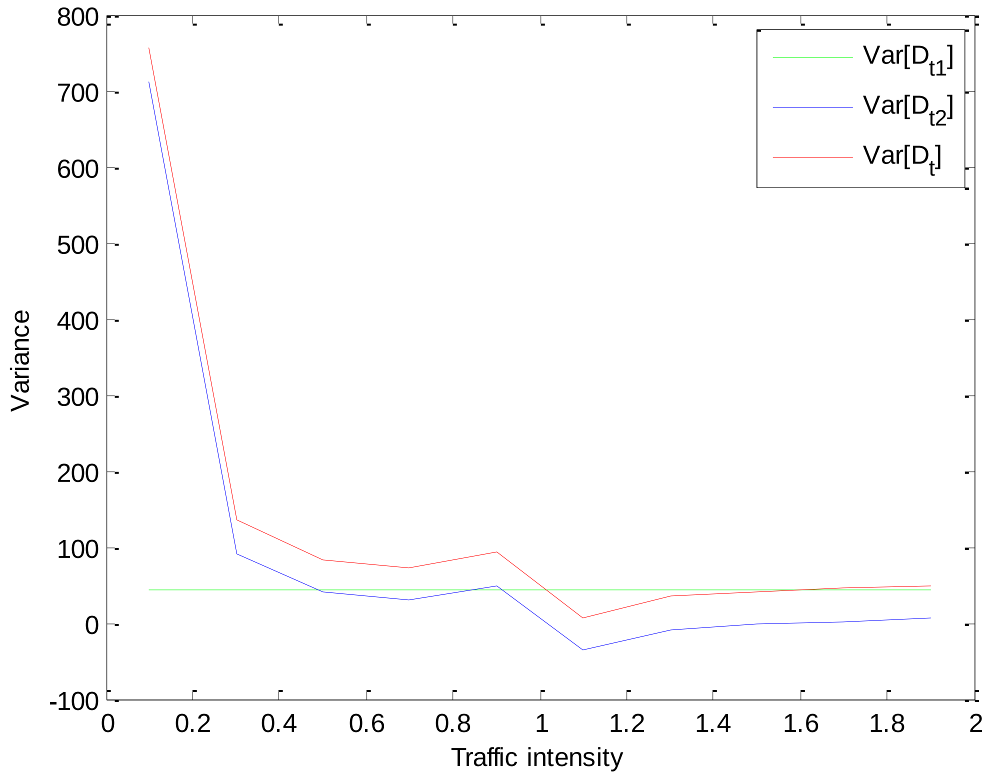

Figure 5.6: $\mathbf{Var\left[D_{t_1}\right], Var\left[D_{t_2}\right]}$ and $\mathbf{Var\ D_t}$ versus ρ using Exponential distribution of service times.

From Figure (5.6), the deterministic model shows no variation because of its constant service times while stochastic model provides a reasonable estimate of variance only under light traffic conditions ($\rho << 1.0$), that is, the variance is time-independent and infinite variance is estimated as ρ approaches 1.0. Therefore, the contributing factor in the estimation of $Var\ D_t$ is D_{t_2} since $Var\left[D_{t_1}\right]$ is zero. A similar scenario is depicted when we assume Gamma distribution for service times as shown in Figure 5.7 below.

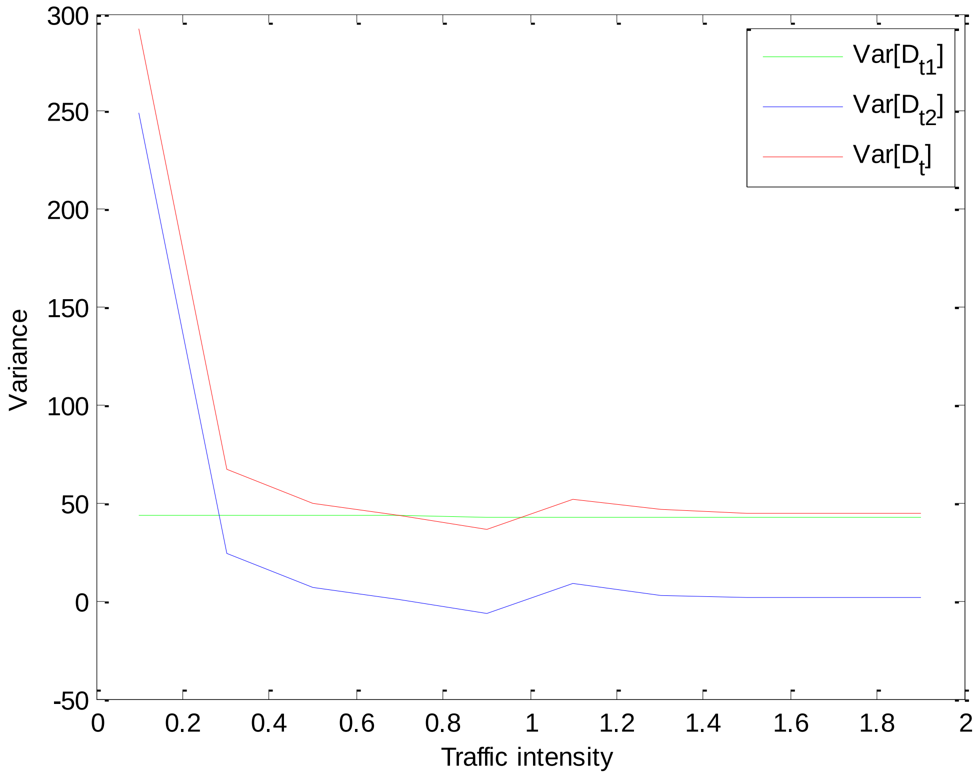

Figure 5.7: $\mathbf{Var}\left[\mathbf{D}_{t_1}\right]$, $\mathbf{Var}\left[\mathbf{D}_{t_2}\right]$ and $\mathbf{Var\ D_t}$ versus ρ using Gamma distribution of service times.

Again, D_{t_2} remains constant as that of exponential distribution of service times due to its deterministic nature of arrivals and service. The stochastic delay component estimates infinite variance when $0.7 \leq \rho \geq 0.9$ contrary to its assumption of steady-state (Hurdle, 1984). This disregards our assumption that D_{t_2} is a steady-state model. Also, Figure 5.7 estimates higher values of $Var\ D_t$ as compared to Figure (5.6), that is, Figure 5.7

estimates 36.74 seconds while Figure 5.6 estimates 7.731 seconds as the lowest values of $Var\ D_t$. Therefore, exponential distribution of service times is far much preferred since a lower variance results to a reduced overall traffic delay at the intersection.

5.5 Application of $\mathbf{Var\ D_t}$

5.5.1 Variability of Level of Service

The possible use of delay variability in quantifying level of service for a signalized intersection is illustrated in this section. In this study, the level of service at the intersection was defined in terms of expected overall traffic delay. With the ability to estimate the variance of overall traffic delay, it is feasible to integrate the concept of reliability into design and analysis of a signalized intersection. For example, delay of a certain percentile, instead of expected value, can be used to define the level of service. A 95th-percentile delay means that 95 percent of the vehicles would encounter a traffic delay less than or equal to this delay. The percentile value can be approximately estimated using $E\ D_t + z_\alpha \sqrt{Var\ D_t}$ where, z_α is a statistic for the normal distribution and can be determined on the basis of the pre-specified reliability level. Figure 5.8 below shows expected overall traffic delay and 90th-percentile delay (with $z_\alpha \approx 1.3$) under different traffic intensities. It is assumed that the ranges of traffic delay values used in defining each level of service in the HCM are also applicable to vehicles, as shown in Figure (5.8). It can be observed that for the given case with a traffic intensity of 0.9, the expected overall traffic delay is 85.6 seconds, which would yield LoS C (point a). However, if the 90th percentile delay is used, the LoS would be D (point b). On the other hand, in order to guarantee that 90 percent of the vehicles going through the intersection encounter LoS C or higher, the traffic intensity needs to be reduced to 0.7 (point c) by either increasing the capacity or reducing the number of arrivals per unit time.

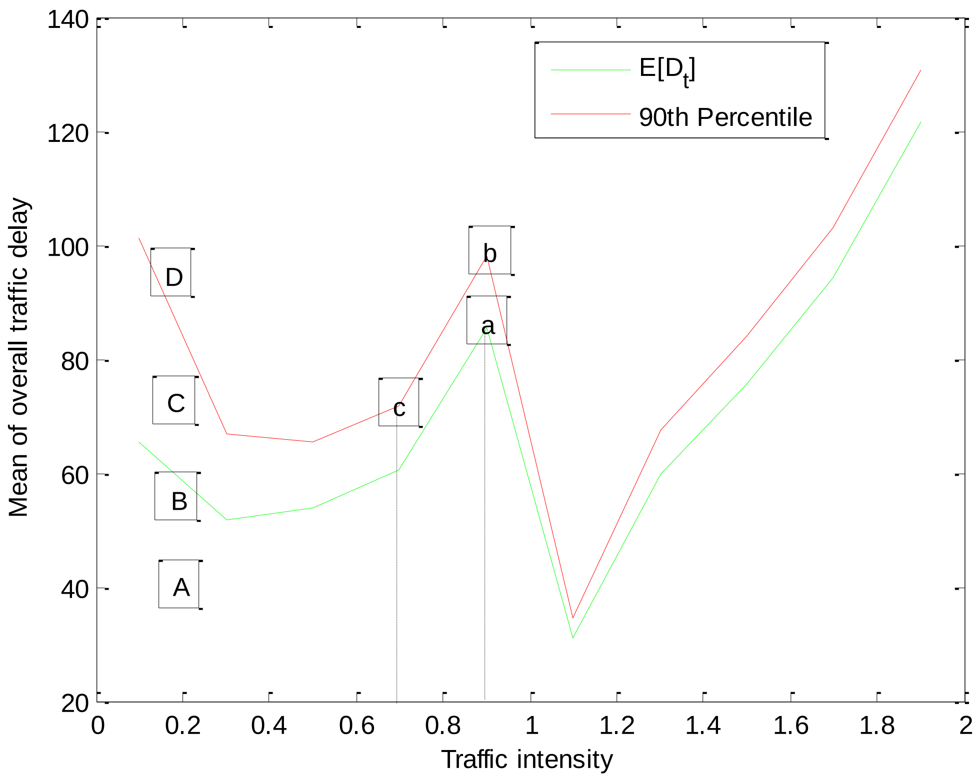

Figure 5.8: $E[D_t]$ and 90th – percentile delay (with $z_\alpha \approx 1.3$) versus ρ.

The MATLAB iteration codes for simulating $E\left[D_{t_1}\right]$, $E\left[D_{t_2}\right]$, $E[D_t]$, $Var\left[D_{t_1}\right]$, $Var\left[D_{t_2}\right]$ and $Var[D_t]$ versus ρ using either Exponential or Gamma distribution of service times are given in Appendix E. These codes were used to plot Figure 5.2 – Figure 5.8. The next chapter provides the conclusion and recommendations of the study.

CHAPTER SIX

CONCLUSION AND RECOMMENDATION

6.1 Introduction

In this chapter, we present the conclusion and recommendations of our study. The conclusion of the study is provided in Section 6.2 while recommendation is provided in Section 6.3.

6.2 Conclusion

Considering the uniform and random properties of traffic flows, the models for estimating deterministic and stochastic delay components of traffic delay were successfully developed in this study. With the application of compressed queueing processes in order to better describe the variation in traffic flows, the developed models indeed estimate the mean and variance of traffic delay at the signalized intersection.

From the developed moments of the deterministic and stochastic delay components of traffic delay, the central moments of the overall traffic delay model were developed. These moments estimate the mean and variance of the overall traffic delay at the signalized intersection.

To validate the developed model, the model was applied to real traffic data collected at Kenyatta Avenue - Kimathi Street intersection and a simulation was performed for traffic intensities ranging from 0.1 to 1.9 using MATLAB software. The simulation results confirmed the result that exists in literature that oversaturated conditions and random delay renders the stochastic model unrealistic. Furthermore, the results preferred exponential distribution of service times to gamma distribution since it resulted to a lower variance hence led to a reduced overall traffic delay.

6.3 Recommendation

In the study presented herein, the overall traffic delay model was developed for a fixed-time traffic light, and further studies should be conducted for vehicle-actuated type of traffic lights.

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APPENDICES

Appendix A: Derivation of (4.4)

$$\int_0^{c_y} \lambda dt \le \int_0^{g_e} \mu dt$$

$$\lambda . t \Big|_0^{c_y} \le \mu . t \Big|_0^{g_e}$$

$$\lambda c_y \le \mu g_e$$

Hence,

$$\frac{\lambda}{\mu} \le \frac{g_e}{c_y}$$

Appendix B: Derivation of (4.20)

$$\mathrm{P}(s) = \pi_0 \sum_{j\ge 0} q_j s^j + \sum_{i\ge 1} \pi_i \sum_{j\ge i-1} q_{j-i+1} s^j$$

$$= \pi_0 Q(s) + \sum_{i\ge 1} \pi_i s^{i-1} \sum_{k\ge 0} q_k s^k \quad , \text{ where } k = j-i+1$$

$$= \pi_0 Q(s) + \sum_{i\ge 1} \pi_i s^i s^{-1} \sum_{k\ge 0} q_k s^k = \pi_0 Q(s) + \frac{1}{s} \sum_{i\ge 1} Q(s)\, \pi_i s^i$$

$$= \pi_0 Q(s) + \frac{Q(s)}{s} \left\{ \sum_{i\ge 0} \pi_i s^i - \sum_{i=0} \pi_i s^i \right\}$$

$$\mathrm{P}(s) = \pi_0 Q(s) + \frac{Q(s)}{s} \left(\mathrm{P}(s) - \pi_0 \right)$$

$$\mathrm{P}(s) = \pi_0 \left(Q(s) - \frac{Q(s)}{s} \right) + \frac{Q(s)}{s} \mathrm{P}(s)$$

$$\mathrm{P}(s) \left(\frac{Q(s)}{s} - 1 \right) = \pi_0 \left(\frac{Q(s)}{s} - Q(s) \right)$$

Hence,

$$P(s) = \pi_0 \frac{Q(s)(1-s)}{Q(s) - s}$$

Appendix C: Derivation of (4.41)

Letting $s = \lambda \; 1 - s$, (4.29) can be re-written as

$$P^* \;\; s \; = \frac{1-\rho \;\; \cdot X^* \;\; s \;\; \cdot s}{\lambda \;\; X^* \;\; s \; - s}$$

Thus,

$$W^* \;\; s \; = \frac{1-\rho \;\; \cdot X^* \;\; s \;\; \cdot s}{\lambda \;\; X^* \;\; s \; - s} \cdot \frac{1}{X^* \;\; s}$$

$$= \frac{1-\rho \;\; \cdot s}{\lambda \;\; X^* \;\; s \; - s}$$

Dividing by s and using the Little's law $\left(\lambda = \dfrac{\rho}{E \;\; X} \right)$

$$W^* \;\; s \; = \frac{1-\rho}{\dfrac{\rho X^* \;\; s}{s \cdot E \;\; X} + 1 - \dfrac{\rho}{s \cdot E \;\; X}}$$

$$= \frac{1-\rho}{1-\rho \left(\dfrac{1 - X^* \;\; s}{s \cdot E \;\; X} \right)}$$

The term between the brackets is the transform of R which is obtained using partial integration method as

$$R^* \;\; s \; = E\left[e^{-sR} \right]$$

$$= \int_{t=0}^{\infty} e^{-sR} f_R \;\; t \;\; dt$$

$$= \frac{1}{E \;\; X} \int_{t=0}^{\infty} e^{-sR} \left[1 - f_R \;\; t \; \right] dt$$

$$= \frac{1}{E \;\; X} \left(\frac{1}{s} - \int_{t=0}^{\infty} \frac{1}{s} \times e^{-sR} f_R \;\; t \;\; dt \right)$$

$$= \frac{1 - X^* \;\; s}{sE \;\; X}$$

Hence,

$$W^*(s) = \frac{1-\rho}{1-\rho R^*(s)}$$

Appendix D: Tables

Table 5.1: Traffic data collected on Wednesday, 20th February, 2013

Time (PM)	t **(sec)**	R **(sec)**	AR **(sec)**	G **(sec)**	Y **(sec)**	l_1 **(sec)**	l_2 **(sec)**	g_e **(sec)**	r **(sec)**	**No. of vehicles passed**
5:13	3600	103	10	52	15	3	4	70	110	73
5:16	3600	104	10	51	15	3	3	70	110	71
5:19	3600	102	10	53	15	4	2	72	108	72
5:22	3600	103	10	52	15	3	4	70	110	75
5:25	3600	101	10	54	15	2	3	74	106	76
5:28	3600	104	10	51	15	3	3	70	110	70
5:31	3600	106	10	49	15	3	4	67	113	70
5:34	3600	107	10	48	15	3	3	67	113	69
5:37	3600	102	10	53	15	4	3	71	109	73
5:40	3600	104	10	51	15	3	4	69	111	71
5:43	3600	97	10	52	15	3	3	77	103	73
5:46	3600	103	10	52	15	3	3	71	109	73
5:49	3600	104	10	51	15	3	3	70	110	70
5:52	3600	103	10	52	15	4	3	70	110	68
5:55	3600	105	10	50	15	3	4	68	112	67
5:58	3600	106	10	49	15	3	2	69	111	69
6:01	3600	106	10	49	15	3	3	68	112	65
6:04	3600	108	10	47	15	2	3	67	113	69
6:07	3600	105	10	50	15	3	4	68	112	66
6:10	3600	107	10	48	15	2	3	68	112	65
	Total							**1396**	**2204**	**1405**

Table 5.2: Traffic data collected on Thursday, 21st February, 2013

Time (PM)	t **(sec)**	R **(sec)**	AR **(sec)**	G **(sec)**	Y **(sec)**	l_1 **(sec)**	l_2 **(sec)**	g_e **(sec)**	r **(sec)**	**No. of vehicles passed**
5:13	3600	105	10	50	15	3	4	68	112	71
5:16	3600	106	10	49	15	3	3	68	112	69
5:19	3600	104	10	51	15	4	2	70	110	70
5:22	3600	105	10	49	15	3	4	68	112	73
5:25	3600	101	10	52	15	2	3	72	106	74
5:28	3600	104	10	49	15	3	3	68	110	68
5:31	3600	108	10	47	15	3	4	65	115	68
5:34	3600	109	10	46	15	3	3	65	115	67
5:37	3600	104	10	51	15	4	3	69	111	72
5:40	3600	106	10	49	15	3	4	67	113	69
5:43	3600	106	10	49	15	3	3	68	112	71
5:46	3600	105	10	50	15	3	3	69	111	71
5:49	3600	106	10	49	15	3	3	68	112	69
5:52	3600	105	10	50	15	4	3	68	112	71
5:55	3600	107	10	48	15	3	4	66	114	70
5:58	3600	108	10	47	15	3	2	67	113	67
6:01	3600	106	10	47	15	3	3	66	114	68
6:04	3600	110	10	45	15	2	3	65	115	65
6:07	3600	107	10	48	15	3	4	66	114	66
6:10	3600	109	10	46	15	2	3	66	114	64
							Total	**1356**	**2240**	**1383**

Table 5.3: Traffic data collected on Friday, 22nd February, 2013

Time (PM)	t (sec)	R (sec)	AR (sec)	G (sec)	Y (sec)	l_1 (sec)	l_2 (sec)	g_e (sec)	r (sec)	No. of vehicles passed
5:13	3600	106	10	49	15	3	4	67	113	65
5:16	3600	105	10	50	15	3	3	69	111	67
5:19	3600	104	10	51	15	4	2	70	110	67
5:22	3600	107	10	48	15	3	4	66	114	64
5:25	3600	104	10	51	15	2	3	71	109	68
5:28	3600	104	10	49	15	3	3	68	110	67
5:31	3600	106	10	48	15	3	4	67	113	65
5:34	3600	109	10	46	15	3	3	65	115	64
5:37	3600	105	10	50	15	4	3	68	112	64
5:40	3600	107	10	48	15	3	4	66	114	64
5:43	3600	107	10	48	15	3	3	67	113	65
5:46	3600	104	10	51	15	3	3	70	110	67
5:49	3600	107	10	48	15	3	3	67	113	65
5:52	3600	106	10	49	15	4	3	67	113	65
5:55	3600	107	10	47	15	3	4	66	114	64
5:58	3600	108	10	47	15	3	2	67	113	65
6:01	3600	109	10	46	15	3	3	65	115	64
6:04	3600	110	10	45	15	2	3	65	115	64
6:07	3600	109	10	46	15	3	4	64	116	63
6:10	3600	108	10	47	15	2	3	67	113	65
							Total	**1342**	**2256**	**1302**

Table 5.4: Average speed per vehicle and distance between the vehicles on the queue

Time (PM)	No. of vehicles passed	Average speed per vehicle (Km/h)	Distance between the vehicles on the queue (Meters)
5:13	65	13	1.2
5:16	67	14	1.4
5:19	67	14	1.3
5:22	64	12	1.3
5:25	68	15	1.2
5:28	67	14	1.4
5:31	65	13	1.3
5:34	64	12	1.4
5:37	64	12	1.4
5:40	64	12	1.3
5:43	65	13	1.2
5:46	67	14	1.2
5:49	65	13	1.1
5:52	65	13	1.3
5:55	64	12	1.3
5:58	65	13	1.3
6:01	64	12	1.4
6:04	64	12	1.4
6:07	63	10	1.1
6:10	65	13	1.2
Total	**1302**	**256**	**25.7**

Appendix E

MATLAB iteration code for simulating $E[D_t]$ versus ρ using Exponential distribution of service times

```
c_y = 180;
g_e = 68.23;
delta = 0.36;
rho = 0.1:0.2:2.0;
mu = 0.38;
lamda = rho.*mu;
E_1 = 1./lamda;
E_2 = 2./(lamda.^2);
%% Mean of the Overall Traffic Delay
%%
ED_t = c_y*((1-g_e/c_y).^2)./(2.*(1-(g_e/c_y).*rho))+...
    (rho.*lamda.*E_2)./(2.*((1-rho).*(1-(lamda.*delta))))+E_1;
figure(5.2)
plot(rho,ED_t,'r');
xlabel('Traffic intensity');
ylabel('Mean of the overall traffic delay');
```

MATLAB iteration code for simulating $E[D_t]$, $E[D_{t_1}]$ and $E[D_{t_2}]$ versus ρ using Exponential distribution of service times

```
c_y = 180;
g_e = 68.23;
delta = 0.36;
rho = 0.1:0.2:2.0;
mu = 0.38;
lamda = rho.*mu;
E_1 = 1./lamda;
E_2 = 2./(lamda.^2);
%%Mean Deterministic Delay Component
%%
ED_t_1 = c_y*((1-g_e/c_y).^2)./(2.*(1-(g_e/c_y).*rho));
%% Mean Stochastic Delay Component
%%
ED_t_2=(rho.*lamda.*E_2)./(2.*((1-rho).*(1-(lamda.*delta))))+E_1;
%% Mean of Overall Traffic Delay
%%
ED_t = c_y*((1-g_e/c_y).^2)./(2.*(1-(g_e/c_y).*rho))+...
    (rho.*lamda.*E_2)./(2.*((1-rho).*(1-(lamda.*delta))))+E_1;
figure(5.3)
plot(rho,ED_t_1,'g');
hold on
plot(rho,ED_t_2,'b');
hold on
plot(rho,ED_t,'r');
xlabel('Traffic intensity');
ylabel('Mean');
legend('E[D_t_1]','E[D_t_2]','E[D_t]');
```

MATLAB iteration code for simulating $E[D_t]$, $E[D_{t_1}]$ and $E[D_{t_2}]$ versus ρ using Gamma distribution of service times

```
c_y = 180;
g_e = 68.23;
delta = 0.36;
rho = 0.1:0.2:2.0;
mu = 0.38;
lamda = rho.*mu;
E_1 = mu./lamda;
E_2 = (mu.*(1+mu))./(lamda.^2);
%%Mean Deterministic Delay Component
%%
ED_t_1 = c_y*((1-g_e/c_y).^2)./(2.*(1-(g_e/c_y).*rho));
%% Mean Stochastic Delay Component
%%
ED_t_2=(rho.*lamda.*E_2)./(2.*((1-rho).*(1-(lamda.*delta))))+E_1;
%% Mean of Overall Traffic Delay
%%
ED_t = c_y*((1-g_e/c_y).^2)./(2.*(1-(g_e/c_y).*rho))+...
    (rho.*lamda.*E_2)./(2.*((1-rho).*(1-(lamda.*delta))))+E_1;
figure(5.4)
plot(rho,ED_t_1,'g');
hold on
plot(rho,ED_t_2,'b');
hold on
plot(rho,ED_t,'r');
xlabel('Traffic intensity');
ylabel('Mean');
legend('E[D_t_1]','E[D_t_2]','E[D_t]');
```

MATLAB iteration code for simulating $Var[D_t]$ versus ρ using Exponential distribution of service times

```
c_y = 180;
g_e = 68.23;
delta = 0.36;
rho = 0.1:0.2:2.0;
mu = 0.38;
lamda = rho.*mu;
E_1 = 1./lamda;
E_2 = 2./(lamda.^2);
E_3 = 6./(lamda.^3);
%%Variance of Overall Traffic Delay
%%
VarD_t = c_y*((1-g_e/c_y).^3)+(1+3.*(g_e/c_y)-4.*(rho.*(g_e/c_y)))...
    ./12.*(1-(g_e/c_y).*rho).^2+3.*((rho.*lamda.*E_2).^2)./4.*((1-
rho).*(1-(lamda.*delta))).^2+...
    ((rho.*lamda.*E_3)./(3.*((1-rho).*(1-(lamda.*delta)))))+...
    (1-(rho.*lamda.*E_1)./(1-rho).*(1-(lamda.*delta))).*E_2-E_1.^2;
figure(5.5)
plot(rho,VarD_t,'r');
xlabel('Traffic intensity');
ylabel('Variance of overall traffic delay');
```

MATLAB iteration code for simulating $\mathrm{Var}\ D_t$, $\mathrm{Var}\left[D_{t_1}\right]$ and $\mathrm{Var}\left[D_{t_2}\right]$ versus ρ using Exponential distribution of service times

```
c_y = 180;
g_e = 68.23;
delta = 0.36;
rho = 0.1:0.2:2.0;
mu = 0.38;
lamda = rho.*mu;
E_1 = 1./lamda;
E_2 = 2./(lamda.^2);
E_3 = 6./(lamda.^3);
%%Variance of Deterministic Delay Component
%%
VarD_t_1 = c_y*((1-g_e/c_y).^3)+(1+3.*(g_e/c_y)-4.*(rho.*(g_e/c_y)))...
    ./12.*(1-(g_e/c_y).*rho).^2;
%%Variance of Stochastic Delay Component
%%
VarD_t_2 = 3.*((rho.*lamda.*E_2).^2)./4.*((1-rho).*(1-
(lamda.*delta))).^2+...
    ((rho.*lamda.*E_3)./(3.*((1-rho).*(1-(lamda.*delta)))))+...
    (1-(rho.*lamda.*E_1)./(1-rho).*(1-(lamda.*delta))).*E_2-E_1.^2;
%%Variance of Overall Traffic Delay
%%
VarD_t = c_y*((1-g_e/c_y).^3)+(1+3.*(g_e/c_y)-4.*(rho.*(g_e/c_y)))...
    ./12.*(1-(g_e/c_y).*rho).^2+3.*((rho.*lamda.*E_2).^2)./4.*((1-
rho).*(1-(lamda.*delta))).^2+...
    ((rho.*lamda.*E_3)./(3.*((1-rho).*(1-(lamda.*delta)))))+...
    (1-(rho.*lamda.*E_1)./(1-rho).*(1-(lamda.*delta))).*E_2-E_1.^2;
figure(5.6)
plot(rho,VarD_t_1,'g');
hold on
plot(rho,VarD_t_2,'b');
hold on
plot(rho,VarD_t,'r');
xlabel('Traffic intensity');
ylabel('Variance');
legend('Var[D_t_1]','Var[D_t_2]','Var[D_t]');
```

MATLAB iteration code for simulating $\mathrm{Var}\ D_t$, $\mathrm{Var}\left[D_{t_1}\right]$ and $\mathrm{Var}\left[D_{t_2}\right]$ versus ρ using Gamma distribution of service times

```
c_y = 180;
g_e = 68.23;
delta = 0.36;
rho = 0.1:0.2:2.0;
mu = 0.38;
lamda = rho.*mu;
E_1 = mu./lamda;
E_2 = (mu.*(1+mu))./(lamda.^2);
E_3 = mu./lamda.^2;
%%Variance of Deterministic Delay Component
%%
VarD_t_1 = c_y*((1-g_e/c_y).^3)+(1+3.*(g_e/c_y)-4.*(rho.*(g_e/c_y)))...
    ./12.*(1-(g_e/c_y).*rho).^2;
%%Variance of Stochastic Delay Component
```

```
%%
VarD_t_2 = 3.*((rho.*lamda.*E_2).^2)./4.*((1-rho).*(1-
(lamda.*delta))).^2+...
    ((rho.*lamda.*E_3)./(3.*((1-rho).*(1-(lamda.*delta)))))+...
    (1-(rho.*lamda.*E_1)./(1-rho).*(1-(lamda.*delta))).*E_2-E_1.^2;
%%Variance of Overall Traffic Delay
%%
VarD_t = c_y*((1-g_e/c_y).^3)+(1+3.*(g_e/c_y)-4.*(rho.*(g_e/c_y)))...
    ./12.*(1-(g_e/c_y).*rho).^2+3.*((rho.*lamda.*E_2).^2)./4.*((1-
rho).*(1-(lamda.*delta))).^2+...
    ((rho.*lamda.*E_3)./(3.*((1-rho).*(1-(lamda.*delta)))))+...
    (1-(rho.*lamda.*E_1)./(1-rho).*(1-(lamda.*delta))).*E_2-E_1.^2;
figure(5.7)
plot(rho,VarD_t_1,'g');
hold on
plot(rho,VarD_t_2,'b');
hold on
plot(rho,VarD_t,'r');
xlabel('Traffic intensity');
ylabel('Variance');
legend('Var[D_t_1]','Var[D_t_2]','Var[D_t]');
```

MATLAB iteration code for simulating $E\left[D_t\right]$ and 90th – percentile delay (with $z_\alpha \approx 1.3$) versus ρ

```
c_y = 180;
g_e = 68.23;
delta = 0.36;
rho = 0.1:0.2:2.0;
mu = 0.38;
lamda = rho.*mu;
E_1 = 1./lamda;
E_2 = 2./(lamda.^2);
E_3 = 6./(lamda.^3);
%% Mean of Overall Traffic Delay
%%
ED_t = c_y*((1-g_e/c_y).^2)./(2.*(1-(g_e/c_y).*rho))+...
    (rho.*lamda.*E_2)./(2.*((1-rho).*(1-(lamda.*delta))))+E_1;
%%Variance of Overall Traffic Delay
%%
VarD_t = c_y*((1-g_e/c_y).^3)+(1+3.*(g_e/c_y)-4.*(rho.*(g_e/c_y)))...
    ./12.*(1-(g_e/c_y).*rho).^2+3.*((rho.*lamda.*E_2).^2)./4.*((1-
rho).*(1-(lamda.*delta))).^2+...
    ((rho.*lamda.*E_3)./(3.*((1-rho).*(1-(lamda.*delta)))))+...
    (1-(rho.*lamda.*E_1)./(1-rho).*(1-(lamda.*delta))).*E_2-E_1.^2;
P=ED_t+1.3.*sqrt(VarD_t);
figure(5.8)
plot(rho,ED_t,'g');
hold on
plot(rho,P,'r');
xlabel('Traffic intensity');
ylabel('Mean of overall traffic delay');
legend('E[D_t]','90th Percentile');
```