**Construction of a Five-Level *V*-Dimensional Modified Third Order Rotatable Designs using a Pair of Pairwise Balanced Designs**

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**Abstract**

*Rotatable designs are devised for use of response surfaces. Rotatability is evidently a greatly popular property for Response Surface Methodology (RSM). RSM is a group of statistical and mathematical techniques valuable for developing, improving and optimizing models and processes. Experimentation of any kind usually requires resources of which they may be limited due to either in availability or high cost of acquisition. To reduce on costs of an experiment one has to make a better preference of the experimental design prior to the experiment itself. An appropriate design that would provide relatively smaller number of the design points of the response at particular points of significance is essential. The aim of this study was to construct modified third order rotatable designs (MTORDs) by use of Pairwise Balanced designs in order to address the problem of the design points. The objective of the study was to construct a five-level v-dimensional modified TORD by using PBD. The five-level v-dimensional modified third order rotatable designs is constructed by use of a suitably chosen pair of Pairwise Balanced Designs by repeating the set of the design points generated from each of the designs a constant number of times. These points were combined together with a number of central points without any additional set of points. In this study, some modified third order rotatable design constructed through pairwise balanced designs were obtained. In conclusion, the modified TORD constructed using PBD yield relatively smaller numeral of the design points as compared to the corresponding existing designs. Other studies that could possibly lead to designs having fewer numeral design points than what is obtained from the present study could be explored. The study recommends further studies on latest methods of construction of modified higher order rotatable designs and applications on this area.*

**Keywords:** Pairwise Balanced Designs, Rotatable Designs, Third Order Rotatable Designs

## INTRODUCTION

Rotatable designs are those devises whose variance of the predictable response at a spot is a function of the distance of that specific point from the source (central point of the design) and thus invariant in orthogonal rotations of the design. Rotatable designs are designed for use of response surfaces. RSM is a group of statistical and mathematical technique valuable for developing, improving and optimizing models and process. Given a response defined by linearity of a function of independent variables, then it's approximating function will be first order model. If a curvature exits in the response, in that case a higher degree polynomial should be used, this leads to second order then to third order and so on till the result expected is attained. These orders were derived due to the need of reducing the cost of experimental design thus the need of having a reduced number of design points. The purpose is to optimize the response (output) influenced by a number of independent variables (input variables). The property of rotatability is a highly desirable quality of an experiment design and was first advanced by Box and Hunter (1957). This property indicates that the variances of estimates of the response made from the least squares estimates of the Taylor series are constant on circle, sphere or hyper spheres on the center of the design. Thus, a rotatable design is a design which achieves this property, it could be rotated through any angle around its center and the variance of responses estimated from it would be unchanged. In these rotatable designs, the moments of independent variable are the same (Box & Hunter, 1957), through order 2d, as those of spherical distribution, or that these moments be invariant under a rotation of the design around the center.

Victorbabu (2009) featured different methods of constructions, among them are; modified response surface designs of order two(2), modified rotatable designs of order two(2), modified SORD with equispaced levels by use of central composite designs, balanced incomplete block designs (BIBD), pairwise balanced designs (PBD), symmetrical unequal block arrangements (SUBA) among other methods. Another paper of Victorbabu (2011) gave a new method of construction of the SOSRD by use (PBD). Tyagi (1964) worked on the constructions of second order and third order rotatable designs using Pairwise balanced (PBD) and doubly balanced designs. Wilson (1972) reviewed in detailed the existence of a theory for a pairwise balanced designs: covering details on the composition theorems and the morphisms. Wilson. (1974) investigated constructions and use of pairwise balanced designs, mathematical centre tracts. Wilson (1975) explored in detail an existence theory for pairwise balanced designs: III, Proof of the existence conjectures. Victorbabu and Rajyalakshmi (2012) constructed a new metghod of Robust slope rotatable designs of second order though pairwise balanced designs. Victorbabu and Surekha (2012) using pairwise balanced designs constructed a measure of second order slope rotatable designs. Dukes and Ling (2014) with prescribed minimum dimension explored pairwise balanced designs. Dukes and Niezen (2015) constructed third dimension of a pairwise balanced designs. Kosgei et al (2013) constructed modified rotatable designs of third order through balanced incomplete block designs. In this study, we are obtaining a modified third order rotatable designs through pairwise balanced designs (PBD) leading to designs with fewer numeral design points than what is given in the existing designs.

**Pairwise Balanced Designs (PBD)**

Pairwise Balanced Design (PBD) is a generalization of a BIBD, in which the blocks may be of different sizes. Take arrangements of *v* treatment with blocks b which we called a PBD of index λ and type $(v, k_1, k_2 \dots k_p)$ given that each block has $k_1, k_2 \dots, k_p$ treatments $(k_i \leq v, k_i \neq k_j)$ and every set of two of distinctive treatments appears in precisely λ blocks of the design. If $k=\{k_1,k_2\dots,k_p\}$ is a set of positive integers, a PBD B[k, λ, v] is a duo (V, B) so as B becomes a collection of Blocks as of v-set of elements so as every pair of elements appears in precisely λ blocks of B and each block B is with cardinality of the set K. Moreover, (X, A) is a usual pairwise balanced design if each point $x \epsilon X$ appears in precisely r blocks $A \epsilon \mathcal{A}$, whereas r is a positive integer. A PBD (X, A) is permitted to include blocks of size |x| (i.e; whole blocks) if (X, A) consists merely of total blocks, it is considered a trivial pairwise balanced design. If (X, A) contains no complete blocks, it is considered a proper pairwise balanced design. A PBD of index λ is a method to select blocks as of a set of treatments in a way that any two treatments have covalence λ. Given that there are V treatment and if each block size is a element of some set of k of positive integers, the design is chosen a PBD (v; k; λ).

## METHODOLOGY

The five-level v-dimensional modified third order rotatable designs was constructed through a suitably chosen pair of Pairwise Balanced Designs by repeating the set of the design points generated from each of the designs a constant number of times. These points were combined together with a number of central points without any additional set of points. The underlying principle behind this modified method considering the case of a pair of PBD is by taking the set of $b_1 2^{2(k_1)}$ design points generated from the first PBD design and repeating a constant number of times, say $n_1$. These points are augmented with the set of $b_2 2^{t(k_2)}$ design points generated from the second PBD design which are again repeated a constant number of times, say $n_2$. The study adopted the approach by Kosgei et al (2013) in construction of modified third order rotatable design using BIBD and applied the same notation as that of Victorbabu (2006) in construction of modified second order rotatable design using pairwise balanced designs (PBD), A review of the known results is given. These results were used to obtain the modified set of moment conditions for the set of points of the design matrix X to form a modified third order rotatable arrangement.

Modified Moments of Rotatable Arrangement of Order 3

The common method of construction a rotatable design of order 3 is to put some limitations demonstrating a number of relations among the moments of third order,

$\sum x_{iu}^2$ , $\sum x_{iu}^4$ , $\sum x_{iu}^2 x_{ju}^2$ , $\sum x_{iu}^6$ , $\sum x_{iu}^4 x_{ju}^2$ , and $\sum x_{iu}^2 x_{ju}^2 x_{lu}^2$ . (1)

The limitations usually used for third order rotatable designs include;

$\sum x_{iu}^4 = 3 \sum x_{iu}^2 x_{ju}^2$ (2)

$\sum x_{iu}^6 = 5 \sum x_{iu}^4 x_{ju}^2 = 15 \sum x_{iu}^2 x_{ju}^2 x_{lu}^2$ (4)

In this study we make use of the following limitations obtained by Kosgei (2013):

$(\sum x_{iu}^2)^2 = N \sum x_{iu}^2 x_{ju}^2$ , i.e. $\lambda_2^2$ (5)

and

$\left(\sum x_{iu}^2 x_{ju}^2\right)^3 = N \left(\sum x_{iu}^2 x_{ju}^2 x_{lu}^2\right)^2$ i.e $\lambda_2 \lambda_6 = \lambda_4^2$ (6)

From these an additional sequence of spherical third order response surface design is obtained providing a relatively fewer number of design points to estimates of the response at particular points of concern compared to what is existing from the related accessible designs.

**Construction of the Designs**

The method of construction a rotatable design of order three both sequential and non-sequential through PBD where $r \neq 3\lambda$ is actualized by taking the *a* – combinations obtained through PBD referred to as *a*-combinations, together with one or more of the combinations of the type (b 0 . . . 0), (c c 0. . . 0), (d d. . . d) Involving fresh unknown levels b, c, and d and then by 'multiplying' them with requisite number of associate combinations. The combinations taken are either the v-combinations obtained from the combination (b 0 . . . 0) by permuting over the different factors, or the combination (d d . . . d) accordingly, as $r < 3\lambda$ or $> 3\lambda$ . The combinations (c c 0 0 . . . 0) give v (v-1)/2 combinations when permuted over all the v factors. The design points obtained by the combination of the type (b 0 . . . 0), (c c 0 . . . 0) and (d d . . . d) after "multiplication" with the requisite associate combinations are denoted respectively as (b 0 . . . 0) X $2^1$, (c c 0 0 . . . 0) X $2^2$ and (d d . . . d) X suitable fraction of $2^v$. According to Das and Narasimham (1962), it becomes necessary sometimes to include the same design more than one set of the same type in order to obtain positive solutions for all the levels.

The current study adopted the method proposed by Victorbabu (2006) for constructing modified second order rotatable design (SORD) using a pair of PBD and applied the conditions of modified third order rotatable designs as obtained by Kosgei et al, (2013). Specific methods for constructing modified TORD for the various levels using a pair of PBD with varied conditions for choosing appropriate designs are given independently while constructing individual designs. The underlying principle behind this modified method considering the case of a pair of PBD is by taking the set of $b_1 2^{2(k_1)}$ design points generated from the first PBD design and repeating a constant number of times, say $n_1$. These points are augmented with the set of $b_2 2^{t(k_2)}$ design points generated from the second PBD design which are again repeated a constant number of times, say $n_2$. $b_i 2^{t(k_2)}$ Denote the number of design points generated from the PBD designs (i = 1,2) by "multiplication", where $2^{t(k_1)}$ and $2^{t(k_2)}$ denote resolution V fractional replicates of $2^{(k_1)}$and $2^{(k_2)}$ factorials with levels ±1. With the above design points together with $n_0$ central points, a modified TORD will be constructed.

## RESULTS AND DISCUSSION

Specific methods for constructing modified TORD for the various levels using a pair of PBD with varied conditions for choosing appropriate designs are given independently while constructing individual designs. The method of constructing of a rotatable design of order three both sequential and non- sequential through PBD where $r \neq 3\lambda$ is actualized by taking the *a* – combinations obtained through PBD referred to as *a*-combinations, together with one or more of the combinations of the type (b 0 . . . 0), (c c 0. . . 0), (d d. . . d) Involving fresh unknown levels b, c and d and then by 'multiplying' them with requisite number of associate combinations

### Five-Level Modified Third Order Rotatable Designs using a Pair of PBD

The method of constructing a modified TORD of five-level by use of a properly chosen pair of pairwise balanced designs (PBD) with no other additional set of points was obtained.
Definition
Let $C_i = (v, b_i, r_i, k_{i1}, k_{i2}, \ldots, k_{ip}, \lambda_i)$, for i=1,2, be an equi-replicated PBD and *v*- number of treatments in an experiment, $b_i$- number of blocks , $r_i$- number of times a treatment is replicated $k_1, k_2 \ldots K_p$- block sizes and $\lambda_i$- number of times each pair is replicated. Let $2^{t(k_i)}$ denote the resolution V fractional replicates of $2^{(k_i)}$ factorials with +1 or -1 levels in treatments with $r_1 \leq 5\lambda_1$ and $r_2 \geq 5\lambda_2$ respectively.
Let $[1 - (v, b_1, r_1, k_{11}, k_{12}, \ldots, k_{1p}, \lambda_1)]2^{t(k_1)}$ and $[a - (v, b_2, r_2, k_{21}, k_{22}, \ldots, k_{2p}, \lambda_2)]2^{t(k_2)}$ denote $b_1 2^{t(k_1)}$ and $b_2 2^{t(k_2)}$ design points generated from PBD through multiplication in that order. The set of $b_1 2^{t(k_1)}$design points obtained from PBD-$C_1$ is repeated $n_1$-times and the set of $b_2 2^{t(k_2)}$design points obtained from the PBD-$C_2$ is repeated $n_2$-times in that order.

Allow $n_o$ to represent the number of mid points. Given the above design points, $n_i b_i 2^{t(k_i)}$, construction of a modified TORD of level five is given in the theorem below.
Theorem 1

The design points
$n_1[1-(v, b_1, r_1, k_{11}, k_{12}, \ldots, k_{1p}, \lambda_1)]2^{t(k_1)} U\, n_2[a-(v, b_2, r_2, k_{21}, k_{22}, \ldots, k_{2p}, \lambda_2)]2^{t(k_2)}$
U $n_o$
Where $vi$- number of treatments in an experiment
$b_i$- number of blocks
$r_i$- number of times a treatment is replicated
$k_{i1}, k_{i2} \ldots K_{ip}$- block sizes
$\lambda_i$- number of times each pair is replicated
and $i = 1,2$
Gives a five level v-dimensional modified TORD in,

$$N = \frac{[n_1 r_1 2^{t(k_1)} + n_2 r_2 2^{t(k_2)} a^2]^2}{[n_1 \lambda_1 2^{t(k_1)} + n_2 \lambda_2 2^{t(k_2)} a^4]} \tag{7}$$

Design points if;

$$(r_1 - 5\lambda_1)(r_2 - 5\lambda_2) \leq 0, \tag{8}$$

$$a^6 = \frac{n_1(r_1 - 5\lambda_1)2^{t(k_1)-t(k_2)}}{n_2(5\lambda_2 - r_2)}, \tag{9}$$

$$\frac{n_2}{n_1} = \frac{(r_1 - 5\lambda_1)2^{t(k_1)-t(k_2)}}{(5\lambda_2 - r_2)} \tag{10}$$

$$n_o = \frac{[n_1 r_1 2^{t(k_1)} + n_2 r_2 2^{t(k_2)} a^2]^2}{[n_1 \lambda_1 2^{t(k_1)} + n_2 \lambda_2 2^{t(k_2)} a^4]} - n_1 b_1 2^{t(k_1)} - n_2 b_2 2^{t(k_2)} \tag{11}$$

And $n_o$ turns out to be an integer.
Proof

In support of the design points obtained, $n_1$-repetitions of points from PBD-$C_1$ and $n_2$-repetitions of points from PBD-$C_2$, for a modified TORD to be true, the conditions are as follows:

$$\sum x_{iu}^2 = n_1 r_1 2^{t(k_1)} + n_2 r_2 2^{t(k_2)} a^2 = X \tag{12}$$

$$\sum x_{iu}^4 = n_1 r_1 2^{t(k_1)} + n_2 r_2 2^{t(k_2)} a^4 = 3Y \tag{13}$$

$$\sum x_{iu}^2 x_{ju}^2 = n_1 \lambda_1 2^{t(k_1)} + n_2 \lambda_2 2^{t(k_2)} a^4 = Y \tag{14}$$

$$\sum x_{iu}^6 = n_1 r_1 2^{t(k_1)} + n_2 r_2 2^{t(k_2)} a^6 = 15Z \tag{15}$$

$$\sum x_{iu}^2 x_{ju}^4 = n_1 \lambda_1 2^{t(k_1)} + n_2 \lambda_2 2^{t(k_2)} a^6 = 3Z \tag{16}$$

Where $X = N\lambda_2$, $Y = N\lambda_4$ and $Z = N\lambda_6$
From (15) and (16), we have,

$$n_1\lambda_1 2^{t(k_1)} + n_2\lambda_2 2^{t(k_2)} a^6 = 5\left[n_1\lambda_1 2^{t(k_1)} + n_2\lambda_2 2^{t(k_2)} a^6\right]$$
(17)
This leads to,
$a^6 = \frac{n_1(r_1-5\lambda_1)2^{t(k_1)-t(k_2)}}{n_2(5\lambda_2-r_2)}$, given in equation (9).
The modified condition $(\sum x_{iu}^2)^2 = N\sum x_{iu}^2 x_{ju}^2$ leads to
$N = \frac{\left[n_1 r_1 2^{t(k_1)} + n_2 r_2 2^{t(k_2)} a^2\right]^2}{\left[n_1\lambda_1 2^{t(k_1)} + n_2\lambda_2 2^{t(k_2)} a^4\right]}$, given in equation (7).
Given $n_o$ central points, we can obtain N directly as follows,
$N = n_1\ b_1 2^{t(k_1)} + n_2 b_2 2^{t(k_2)} + n_o$.
(18)
Example 1

The design points
$n_1[a - (v = 9, b_1 = 13, r_1 = 4, k_{11} = 4, k_{12} = 3, k_{13} = 2, \lambda_1 = 1)]2^4 U\ n_2[a - (v = 9, b_2 = 30, r_2 = 7, k_{21} = 3, k_{22} = 2, \lambda_2 = 1)]2^3$

U $n_o$
Gives a five-level 9-dimensional modified rotatable designs of order three with N= 1200 design points as obtained from equation (7) with $n_1 = 2,\ n_2 = 2$ respectively as obtained from (10). In this case (9) gives $a^6$=1 and (11) gives $n_o$ = 304
Consider two pairwise balanced designs, PBD-$C_1$
$(v = 9, b_1 = 13, r_1 = 4, k_{11} = 4, k_{12} = 3, k_{13} = 2, \lambda_1 = 1)$ and PBD-$C_2$
$(v = 9, b_2 = 30, r_2 = 7, k_{21} = 3, k_{22} = 2, \lambda_2 = 1)$. The design sets can be represented as incident matrix as follows:

Let *inc.* PBD-$C_1$ $(v = 9, b_1 = 13, r_1 = 4, k_{11} = 4, k_{12} = 3, k_{13} = 2, \lambda_1 = 1)$be given as follows;

Incidence Matrix 1

| $t_1$ | $t_2$ | $t_3$ | $t_4$ | $t_5$ | $t_6$ | $t_7$ | $t_8$ | $t_9$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |

This can as well be written in block form as:

1 2 3 4
1 5 6
2 5 7
3 5 8
4 5 9
1 7 9
2 8 9
3 6 9
4 7 8
1 8
2 6
3 7
4 6

Using the relation, $\sum x_{iu}^{6} = 5\sum x_{iu}^{2}x_{ju}^{4}$ , we obtain,
$4 = 5 \times 1]2^{4}$ design points.
$\Rightarrow 64 = 80$

*a(i)* (19)

Let *inc.* PBD-C2 $(v = 9, b_2 = 30, r_2 = 7,\ k_{21} = 3, k_{22} = 2, \lambda_2 = 1)$be given as follows;
Incidence Matrix 2

| t1 | t2 | t3 | t4 | t5 | t6 | t7 | t8 | t9 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | | |

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 1 | | | | | |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 0 | | | | | |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | | | | | |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | | | | | |
| 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | | | | | |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | | | | | |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | | | | | |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | | | | | |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 0 | | | | | |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | | | | | |

This can as well be written in block form as:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 2 3 | 14 | 1 7 | 2 4 | 2 7 | 3 4 | 3 7 | 4 7 | 5 7 | 6 7 |
| 4 5 6 | 1 5 | 1 8 | 2 5 | 2 8 | 3 5 | 3 8 | 4 8 | 5 8 | 6 8 |
| 7 8 9 | 1 6 | 1 9 | 2 6 | 2 9 | 3 6 | 3 9 | 4 9 | 5 9 | 6 9 |

Using the relation, $\sum x_{iu}^6 = 5\sum x_{iu}^2 x_{ju}^4$ , we obtain,

$7 = 5 \times 1]2^3$ design points.

$\Rightarrow 56 = 40$

*a(ii)* (20)

Repeating the set of points $C_1$, (*a(i)*) (19), two times and in design $C_2$, (*a(ii)*) (20), two times (since n1= 2 and n2= 2), we obtain,

$C_1$ – $[64 = 80] \times 2$ $\qquad$ $128 \leftrightarrow 160$

\+ $C_2$ - $[56 = 40] \times 2$ $\qquad \Rightarrow$ $112 \leftrightarrow 80$

240 = 240

(21)

From the above it is crystal clear that the number of design points obtained is much less than the number of design points earlier obtained, that is, the design points earlier obtained is N=1200 as obtained from equation (7) and the designs points obtained after is N=240 from (21). This shows a great reduction in the number of design points thus it cuts on costs of experimentation.

**Summary of Results**

A five-level modified third order rotatable design was constructed using a suitably chosen pair of pairwise balanced designs (PBD) without any additional set of points. In example 1, the number of design points obtained is much less than the design points earlier obtained i.e final N=240 design points from N=1200 design points. This showed that, the set of the design points;

$n_1[1-(v, b_1, r_1, k_{11}, k_{12}, \ldots, k_{1p}, \lambda_1)]2^{t(k_1)} U n_2[a-(v, b_2, r_2, k_{21}, k_{22}, \ldots, k_{2p}, \lambda_2)]2^{t(k_2)}$
U $n_o$ gives a five level v-dimensional modified TORD in,
$N = \frac{[n_1 r_1 2^{t(k_1)} + n_2 r_2 2^{t(k_2)} a^2]^2}{[n_1 \lambda_1 2^{t(k_1)} + n_2 \lambda_2 2^{t(k_2)} a^4]}$ , design points, under specified restrictions. Refer to (7)

Or
$N = n_1 b_1 2^{t(k_1)} + n_2 b_2 2^{t(k_2)} + n_o$. When given $n_o$ central points. Refer to (19)
Thus, the method of construction of a five-level modified third order rotatable design was constructed using a suitably chosen pair of pairwise balanced designs (PBD) without any additional set of points. This gave a modified third order rotatable designs with a smaller number of design points constructed through pairwise balanced designs, as compared to existing designs of the same dimensions hence they are cost effective.

## CONCLUSION AND RECOMMENDATION

### Conclusion

The construction of rotatable designs using pairwise balanced designs has been studied by a number of authors in the previous years. A five-level modified third order rotatable design was constructed using a suitably chosen pair of pairwise balanced designs (PBD) without any additional set of points. A design with fewer number of design points relative to the existing corresponding design (For instance, in Example 1, design points' reduction from N=1200 to N= 240) was obtained. The implications of fewer number of design points leads to effective and reduced cost of experimentation.

### Recommendations

The study recommends establishing suitable and appropriate areas of practical applications of the designs obtained.

Other studies that could possibly lead to designs with fewer number of design points than what is obtained in the present study could be explored

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